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POPULAR
AND
MATHEMATICAL
ASTRONOMY.

POPULAR AND MATHEMATICAL ASTRONOMY.

POPULAR AND MATHEMATICAL ASTRONOMY,

WITH THE PRINCIPAL FORMULÆ

OF

PLANE AND SPHERICAL TRIGONOMETRY,

FOR THE USE OF SCHOOLS.

BY

WILLIAM T. READ,

LATE OF THE ROYAL NAVAL SCHOOLS, GREENWICH.

LONDON:
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PREFACE.

In the First Part of this work I have endeavoured to set before the beginner of Popular Astronomy the theories connected with that sublime science in such a manner that he may most easily retain them.

The Second Part contains the proofs of the formulæ on which the investigations following depend, and is arranged in such a manner as to be rendered profitable to any one who possesses a fair knowledge of Euclid, Algebra, &c.

I therefore lay this work before my fellow labourers in education, earnestly hoping that it may prove useful.

W. T. R.

Chiswick,
November 1861.

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PART I.

POPULAR ASTRONOMY.

PART I.

POPULAR ASTRONOMY.

INTRODUCTORY CHAPTER.

1. Astronomy is the science which treats of the motions of the heavenly bodies, and the different appearances they present.

2. By the heavenly bodies we mean the Sun, planets, comets, and fixed stars.

3. The Sun is the centre round which the planetary system revolves; and they compose what is called the solar system.

4. Planets are of two kinds, primary and secondary.

5. Primary planets are those which move regularly round the Sun, and receive light from him.

6. Secondary planets are those which revolve round the primary ones.

7. Fixed stars are those bodies which shine by their own light, and are not subject to any perceptible motion.

8. A number of stars apparently near each other, which may be conceived to form a figure of some particular shape, is called a constellation.

9. Opaque bodies like the planets, moving in their own orbits, and having a luminous train, are called comets.

10. The position which a planet has when seen from the centre of the earth, is called its geocentric place.

11. The position which a planet would have, if seen by a spectator in the centre of the Sun, is called its heliocentric place.

12. The angular apparent distance of a planet from the Sun, as seen from the Earth, is called its elongation.

13. The apparent change of place in the fixed stars, arising from the motion of the earth, and that of light, is called aberration.

14. When a celestial body passes before another, so as to make any part invisible, it is said to make a transit.

15. The orbit of a planet is the curve it describes in its revolution round the Sun.

16. The shape of an orbit is not circular, but oval or elliptical, so that planets are nearer to the Sun at some periods than at others.

17. Those planets which are without the earth's orbit, are called superior planets ; those within it, inferior.

18. The planets revolve round the Sun in the following order :—Mercury, Venus, the Earth, Mars, the Asteroids, Jupiter, Saturn, Herschel or Uranus, and Neptune.

QUESTIONS ON THE INTRODUCTORY CHAPTER.

1. Give a definition of astronomy ?
2. What is meant by the heavenly bodies ?
3. What is the solar system ?
4. Into what classes are planets divided ?
5. Define the first of these ?
6. What are secondary planets ?
7. What are fixed stars ?
8. Define a constellation ?
9. What are comets ?
10. What is the geocentric place of a planet ?
11. What is its heliocentric place ?
12. What is meant by elongation ?
13. Give a definition of aberration ?
14. When does a body make a transit ?
15. What is the orbit of a planet ?
16. What is its shape, and the consequences resulting ?
17. What are superior and inferior planets ?
18. Name the planets of the solar system ?

CHAPTER II.

THE SUN.

1. The Sun is the centre of the system which is called after his name, the planets of which all receive a greater or less degree of light and heat from him. He has been called by Arago, (the celebrated French astronomer,) "a star spread among stars;" by Copernicus, "the torch of the universe."

2. The greater part of the ancients regarded him as a flaming body; but he is now considered to be opaque like the earth, diffusing light by his luminous atmosphere.

3. It is considered by some of the present day, that the space around the Sun is filled up by an elastic and rarified substance called "ether." This, by a rapid vibrating movement, produces upon the eye the phenomenon of light, in the same manner as that in the air produces sound.

4. The Sun as seen through a telescope presents some places less bright than others on its disc, and of different forms. After an interval of 12 or 13 days, new places appear, which may be again seen after a certain lapse of time. In 1637 they were so numerous, that the heat and light were sensibly diminished.

5. The first who supposed the Sun to have a rotation on his axis, was Jordan Bruno, who published a treatise on the universe in 1591.

6. From the appearances of these spots we see that he turns on his axis in 25 days: we are not yet acquainted with the nature of these spots.

7. According to Herschel, Laplace, and others, the Sun is surrounded by a luminous atmosphere.

8. "With regard to the Sun's being inhabited," says Arago, "if any one were to put to me this simple question, I should reply that I did not know, but if asked whether he could be inhabited, I should reply in the affirmative."

9. Before the time of Copernicus, it was believed that the Sun was a body revolving round the Earth, and that the latter was stationary. He it was who established the system, called

after him the Copernican system, and which continues to the present time.

10. It has been discovered that the Sun is in size 1,300,000 times that of the Earth.

11. If the Sun were a body of fire, it would follow that the higher we might go above the surface of the Earth, the hotter it would be; but it is a wêll-known fact that the cold increases, for the tops of high mountains are always covered with snow and ice.

12. Heat is generated from the luminous atmosphere, by its coming into contact with that of the planets; for heat may be produced by the mixture of several chemical fluids. If a piece of flint and steel be struck together, heat is produced, although each is a cold body.

13. The Sun's diameter is about 950,000 miles, and his distance from us varies at different seasons, for the Earth's orbit is an ellipse. This causes his apparent diameter to vary, and his bulk to appear greater at some periods than at others.

14. He turns on his axis (which is perpendicular to the plane of the Earth's orbit), in the same direction as the Earth and the other planets revolve about him, viz., from west to east.

15. It has been calculated that a locomotive engine, travelling at the rate of 39 miles an hour, would require 342 years to perform the journey to this immense luminary; and a cannon ball, which travels with a velocity 50 times greater, would require 7 years to go the same distance.

16. The planets, with their satellites, if combined, would make a globe of only the 150th part in size of the Sun.

QUESTIONS ON CHAPTER II.

1. What is the Sun? What has he been called?
2. What was the supposition of the ancients?
3. Explain how the phenomenon of light is caused.
4. What are the spots on the Sun? Give an instance of their effect.
5. Who first supposed the Sun to move on his axis?
6. How long is he in turning on his axis?
7. Has he any atmosphere?
8. What does Arago say in reference to his being inhabited?
9. What did Copernicus prove?
10. Of what size is the Sun?

11. Prove that he is not a body of fire.
12. How is heat generated? Give an illustration.
13. What is his diameter? Is his distance constant?
14. In what direction does he turn on his axis?
15. Give a proof of his great distance.
16. Give another proof of his immense size.

CHAPTER III.

MERCURY (☿).

1. Mercury is the nearest planet to the Sun, round which he is very rapidly carried.

2. His distance from the Sun is 37,000,000 miles; therefore that luminary would appear to a spectator in this planet nearly three times as large as from the Earth.

3. He performs his orbit in 88 days, and moves at the rate of 95,000 miles per hour.

4. The time of his rotatory motion and the inclination of his axis not having been yet ascertained, we are ignorant of the lengths of day and night, and the variety of his seasons.

5. He is the smallest of the primary planets, and, in consequence of his proximity to the Sun, is seldom rendered visible, being lost in the brightness of the rays.

6. He is best observed when at his transit over the Sun's disc, at which time he may be seen through a telescope, as a dark spot on its surface.

7. He may be oftenest seen by the naked eye in the southernmost parts of the Earth.

8. If we consider the Sun to be fire, then the amount of light and heat would be seven times as great as that on the Earth, or sufficient to make water boil. It may, however, be less intense, since the Sun is generally considered as an opaque body, diffusing light by his atmosphere.

9. This planet never appears quite full, for being within the Earth's orbit, his enlightened side is turned towards the Sun.

10. On account of his amazing rate of travelling, he has received from the Greeks the name of Mercury, the messenger of the gods.

QUESTIONS ON CHAPTER III.

1. What is the position of Mercury in the solar system?
2. How far distant is he from the Sun?
3. How long is he in performing his orbit? At what rate?
4. Why are we ignorant of the lengths of his day and night?
5. Why is he so seldom visible?
6. When is he best observed?
7. Where may he be oftenest seen?
8. Is it probable that his amount of light and heat are intense?
9. Why is it that he never appears full?
10. From whom did he receive his name, and why?

CHAPTER IV.

VENUS (♀).

1. This, the next in order, is the other inferior planet, but the brighter, and apparently the larger.

2. Her diameter is nearly 8,000 miles, and her distance from the Sun 68,000,000 miles.

3. She performs her orbit in 224⅔ days, and moves at the rate of 81,000 miles per hour, and is supposed to turn on her axis in 23 hours 21 minutes.

4. When she is west of the Sun, and consequently rises before him, she is a morning star, and is called Phosphorus or Lucifer.

5. When she appears east of the Sun, and therefore sets after him, she is an evening star, and is called Hesperus or Vesper.

6. She is alternately a morning and evening star 290 days.

7. Since the Earth revolves the same way, though not with such swiftness as this planet, she is seen east or west of the Sun for a longer time than she requires to perform her orbit.

8. Venus has phases like those of the Moon, in consequence of her being one of the inferior planets; and her illuminated side is constantly directed to the east when a morning star, and to the west when an evening one.

9. Spots have been seen on her surface, similar to those on the Moon, and her mountains are supposed to be five or six times as high as those on the Earth.

QUESTIONS ON CHAPTER IV.

1. To what class of planets does Venus belong ?
2. What is her diameter and distance from the Sun ?
3. How long is she in performing her orbit and turning on her axis ?
4. When is she a morning star ?
5. When is she an evening star ?
6. How long does she continue to be each ?
7. What is the consequence of the Earth's revolving the same way ?
8. Has she any phases ?
9. What do you know of the size of her mountains ?

CHAPTER V.

THE EARTH (⊕).

1. The world on which we live is the third planet from the Sun.

2. The shape of the Earth is nearly round ; it is a little flattened at the poles, the equatorial diameter being about 26 miles longer than the polar one.

3. It performs its orbit in 365¼ days, and turns on its axis once in 24 hours.

4. Its distance from the Sun is 95,000,000 miles, the polar diameter is about 7,916, and that at the equator 7,942 miles.

5. A figure similar to that of the Earth is called an oblate spheroid, or flattened sphere.

6. The rotundity of the Earth is proved from the following:—1st, it is observed that to whatever quarter a ship may sail, after she has proceeded a few miles to sea she is gradually lost to view, appearing to sink behind the convexity of the waters ; 2ndly, in eclipses of the Moon (caused by the Earth's coming between her and the Sun), the shadow cast on the Moon has always a curved outline.

7. The imaginary line passing through the centre of the Earth, from one flattened part to the other, is called the axis.

8. The extremities of the axis are called the north and south poles.

9. An imaginary line, or rather circle, drawn round the Earth, at an equal distance from each pole, is called the equator.

10. The coldest parts of the Earth are within what are called the Arctic and Antarctic circles, and the hottest parts extend about 23½ degrees on each side of the equator.

11. The two remaining parts have a temperate climate.

12. The Earth being a globe, only one half of it can be enlightened by the Sun at the same time, so that when it is day to one half it is night to the other : the turning of the Earth on its axis, therefore, produces day and night.

13. The Earth's axis is inclined to the plane of its orbit, at an angle of 23½ degrees, and always points to the pole star, which may be considered as a fixed point in the heavens.

14. In consequence of this, as the Earth performs its orbit, we have the seasons, spring, summer, autumn, and winter, in succession.

15. In March and September neither pole is turned to the Sun, so that we have equal day and night throughout the Earth.

16. These two periods of the year are called the vernal and autumnal equinoxes respectively. The word equinox is derived from the Latin *æquus* (equal), and *nox* (night).

17. In June the north pole is turned to the Sun : we have then long days and short nights. This period is called the summer solstice, from *sol* (the sun), and *stare* (to stand), because the Sun appears to be in the same place at mid-day for several days together.

18. In December the south pole is turned to the Sun : we have then short days and long nights.

19. The Earth's orbit being an ellipse, we are much nearer the Sun at one time than at another ; we are nearest in December.

20. An ellipse is a figure which has two particular points in its major or greater diameter, which if brought together, the ellipse becomes a circle ; these points are called the foci.

21. In the orbit of the Earth the Sun occupies one of these.

22. The Earth always turns on its axis in the same direction, viz., from west to east ; this causes the Sun to appear to move in the heavens from east to west.

23. Since the atmosphere and everything on the Earth's surface are carried round with it, we do not perceive its motion.

24. The Earth possesses an attraction called gravitation, which draws everything towards it, acting on all bodies as a magnet does on iron.

Questions on Chapter V.

1. What is the Earth's position in the solar system.
2. Of what shape is the Earth?
3. How long is it in turning on its axis and performing its orbit?
4. What is our distance from the Sun, and the lengths of the Earth's diameters?
5. What name is given to the figure of the Earth?
6. Prove its rotundity.
7. Define the Earth's axis.
8. What are the poles?
9. What is the equator?
10. Where are the coldest and hottest parts?
11. What parts have a temperate climate?
12. Explain the cause of day and night.
13. What is known of the Earth's axis?
14. Why do we have the seasons in regular succession?
15. What position has the Earth in spring and autumn?
16. What are these periods called?
17. What position has the Earth in June? What is the meaning of solstice?
18. In what position is the Earth in December?
19. When are we nearest to the Sun?
20. Explain the ellipse.
21. What position has the Sun in the Earth's orbit?
22. Why does the Sun appear to move from east to west?
23. Why do we not perceive the Earth's motion?
24. Has the Earth any attraction?

CHAPTER VI.

THE MOON (☾).

1. The Moon is the satellite which attends the Earth, in its yearly path round the Sun. She revolves about the Earth in an elliptical orbit.

2. Her diameter is about 2,200 miles, or a little more than a quarter of that of the Earth.

3. Her distance from the Sun is about 95,000,000 miles, and from the Earth 240,000.

4. She revolves round the Earth in the same time that she turns on her axis, viz., one lunar month (29 days 12 hours), and therefore she has only one day and night in a month.

5. She appears larger than the inferior planets on account of her proximity to the Earth ; and next to the Sun she is the most splendid of all the heavenly bodies.

6. She receives her light from the Sun, and is not surrounded by a luminous atmosphere as that body : if it were so she would always appear full ; whereas she presents horned, semicircular, and other appearances.

7. When only a small part of her enlightened side is turned to the Earth, she presents a horned appearance.

8. This happens a few days before and after a new moon.

9. When more than half of her enlightened side is turned to us, she is said to be gibbous.

10. When she is between the Earth and Sun, her dark side is turned to the former ; it is then new moon.

11. When the Earth is between the Sun and Moon, her illuminated side is turned to us ; it is then full moon.

12. From the same spots on her disc being always visible, it is known that she always presents the same face to the Earth.

13. The reason of this is, that the time of rotation about the Earth and on her axis is the same.

14. The appearance of the Earth to the inhabitants of the Moon must be much the same as that of the Moon to the Earth, excepting that it must appear about thirteen times as large to them, as the Moon to us.

15. The several islands and continents must resemble spots, which by their appearing and disappearing would afford her inhabitants an accurate measure of time.

16. A lunar month is the interval between two new moons.

17. A periodic month is the time she takes to describe her orbit ; it is shorter than a lunar month.

18. The physical cause of her motion round the Earth is the mutual attraction of the Earth and Moon.

19. The attraction of the Sun on the Moon is greater than that of the Earth on her ; this causes an irregularity in her motion, called her retardation.

20. The face of the Moon, when viewed through a telescope, is greatly diversified, some parts being brighter than others.

21. These are evidently hills and valleys, which is confirmed by her edge appearing jagged or rough, especially when she is gibbous.

22. We perceive by the shadow of the spots that they are mountains; for the elevated parts always cast a triangular shadow, in a direction from the Sun; and the cavities next that body are dark, and illuminated on the opposite side.

23. As the mountains are of considerable elevation, they are often illuminated at considerable distances from the confines of the enlightened hemisphere, and thus afford a measure of determining their height.

24. They are, in proportion, much higher than those on the earth : there is one, called St. Catherine, supposed to be nine miles high.

25. Some of the valleys are from 9,000 to 18,000 feet in depth ; the width of some is 15 miles.

26. It is generally supposed that there are no oceans nor seas in the Moon.

27. In 1787, Dr. Herschel discovered three volcanoes ; two seemed almost extinct, but the third showed an actual eruption of luminous matter, resembling pieces of burning charcoal.

28. In March, 1794, Mr. Wilkins, of Norwich, observed another very bright spot on the surface of the Moon, which continued visible for about five minutes.

29. It was formerly supposed that the Moon had no atmosphere, but it has been discovered that she has, though not above one-third of the density of ours.

30. The full Moon of harvest-time is bright, because she rises more immediately after the Sun at that time than at any other part of the year.

31. An eclipse of the Moon is caused by the Earth's shadow falling on her, and happens only when she is in opposition to the Sun or at the full.

32. An eclipse does not happen at every full moon; for the orbit being inclined, she is not always in a straight line with the Earth and Sun, but either above or below it; consequently the Earth's shadow does not fall on the Moon.

33. A total eclipse of the Sun can only happen when the Moon is in her nodes.

34. The nodes are those two points in which her orbit cuts the ecliptic.

35. The ecliptic is so called because eclipses of the Moon can only happen when she is in this circle.

36. The reason of her rising more immediately after the Sun at harvest-time than at any other time of the year, is, that she is full when in the signs Pisces and Aries, and those signs rise in a shorter time than others.

37. In winter the Moon is in these two signs during her first quarter; in spring, about the time of her change, when she rises with the Sun; in summer, about the time of her last quarter, when she rises at midnight, and therefore in autumn she is full only in those signs, and rises at sunset.

38. She dissipates in some degree the darkness of night, subdivides the year into months, and regulates the tides.

Questions on Chapter VI.

1. What is the Moon? Of what shape is her orbit?
2. What is her diameter?
3. How far distant is she from the Earth and Sun?
4. How long is she in turning on her axis?
5. Why does she appear large?
6. How do we know that she receives her light from the Sun?
7. What is the cause of her appearing horned?
8. When does this happen?
9. When is she gibbous?
10. What is a new moon?
11. What is her position when full?
12. How do we know that she always presents the same face to the Earth?
13. What is the cause of this?
14. How must the Earth appear to her inhabitants?
15. In what way can they have an accurate measure of time?
16. What is a lunar month?
17. What is a periodic month?
18. What is the cause of her motion round the Earth?
19. What causes an irregularity in it?
20. Describe her appearance through a telescope?
21. Prove that she has hills and valleys?
22. What proof have we of the existence of mountains?
23. How has their height been determined?
24. What is the name of the highest mountain?
25. What is the depth and width of some of the valleys?

26. Has she any oceans or seas?
27. Who first discovered her volcanoes?
28. What was seen in 1794?
29. Has she any atmosphere?
30. Why is the full moon of harvest-time bright?
31. How is an eclipse caused? When does it happen?
32. Why does not an eclipse happen at every full moon?
33. When does a total eclipse of the Sun happen?
34. What are the nodes?
35. Why is the ecliptic so called?
36. Why does she rise sooner at harvest time?
37. Is she in Pisces and Aries only during harvest?
38. Name the uses of the Moon?

CHAPTER VII.

MARS (♂).

1. Mars is the least bright of all the planets; he is of a fiery, reddish hue, and from the duskiness of his appearance is supposed to be surrounded by a dense, cloudy atmosphere.

2. His diameter is 4,200 miles, and therefore he is much less than either Venus or the Earth.

3. He moves between the orbit of the Earth, and that of the asteroid Ceres.

4. His distance from the Sun is 144,000,000 miles, and the length of his year 687 days.

5. He turns on his axis in 24 hours 39½ minutes.

6. In consequence of his deriving light from the Sun, and revolving about that body, he has an increase and decrease like the Moon.

7. He is almost bisected when in the quadrature, but never horned, like the inferior planets.

8. No difference in the length of his diameter has ever been discovered; consequently he is supposed to be of a perfectly spherical form.

QUESTIONS ON CHAPTER VII.

1. What is the appearance of Mars?
2. What is his diameter?
3. Where is his orbit placed?
4. What is his distance from the Sun, and the length of his year?
5. How long is he in turning on his axis?
6. Why has he an increase and decrease like the Moon?
7. What appearances does he present?
8. Of what shape is this planet?

CHAPTER VIII.

THE ASTEROIDS.

1. The principal of these bodies are Ceres, Pallas, Juno, and Vesta.

2. The first of these, Ceres, was discovered at Palermo, in January 1801, by Mr. Piazzi.

3. Her diameter is, according to most astronomers, 1,624 miles.

4. Her distance from the Sun is 260,000,000 miles.

5. She performs her orbit, which is immediately outside that of Mars, in 4 years 221 days.

6. The second, Pallas, was discovered in March 1802, by Dr. Olbers, at Bremen.

7. Her diameter, is supposed to be about 3,000 miles.

8. Her distance from the Sun is 266,000,000 miles, and she performs her orbit in 4 years 7 months.

9. Her orbit is next to that of Ceres, which crosses it. Pallas comes nearer the Sun at her perihelion than Ceres, and goes to a greater distance at her aphelion.

10. Juno was discovered in September 1804, near Bremen, by Mr. Harding.

11. Her diameter is 1,420 miles, and her distance from the Sun 300,000,000 miles.

12. She performs her orbit, which is next that of Pallas, in 4½ years.

13. Vesta was discovered in March 1807, by Dr. Olbers.

14. Her magnitude and distance have not yet been truly ascertained ; she performs her orbit in 3 years and 66 days.

15. She moves between the orbits of Juno and Jupiter.

16. Some of the other asteroids are Astræa, Hēbe, Iris, Flora, Mētis, Hygeia, Parthenope, Victoria, Egēria, Irēne, Eunōmia, Psyche, Thētis, Melpŏmene, Fortūna, Massĭlia, Lutētia, Calliope, Proserpine, Bellōna, Amphitrite, and Urānia.

17. The astronomical characters of these are, Ceres (⚳), Pallas (⚴), Juno (⚵), Vesta (⚶). The rest are distinguished by numbers, beginning with Astræa (5) and ending with Urania (30.)

QUESTIONS ON CHAPTER VIII.

1. Name the principal asteroids.
2. When was the first discovered, and by whom?
3. What is her diameter?
4. What is her distance from the Sun ?
5. How long is she in performing her orbit ?
6. When was the second discovered, and by whom ?
7. What is her diameter ?
8. What is her distance from the Sun, and how long is she in performing her orbit ?
9. Where is her orbit placed ?
10. When was the third discovered, and by whom ?
11. What is her diameter, and distance from the Sun ?
12. How long is she in performing her orbit ?
13. When was the fourth discovered ?
14. How long is she in performing her orbit ?
15. Where is it placed ?
16. Name some of the other asteroids ?
17. Write down the astronomical characters of these.

CHAPTER IX.

JUPITER (♃).

1. This is the largest of all the planets, and is easily distinguished from the rest by his particular light.

2. His diameter is 89,200 miles, and his magnitude one thousand times that of the Earth.

3. He performs his revolution in 11 years 314½ days, moving at the rate of 25,000 miles an hour, and revolves on his axis in 9 hours 55 minutes.

4. The most remarkable feature of this planet is that he is surrounded by faint belts, which change so much that they have been ascribed to islands. Some of them have been interrupted and broken, and have vanished entirely. Large spots have also been seen on these belts, and when one vanishes the adjoining spots disappear with it.

5. His axis is so nearly perpendicular to the plane of his orbit that he has no difference of season.

6. If it were much inclined, vast tracts round the poles would not have any of the Sun's light for years together.

7. The difference between the polar and equatorial diameters is 6,000 miles.

8. The Sun must appear to the inhabitants of this planet 48 times less than to us, and the light must be less in proportion.

9. The deficiency of light is supplied partly by the shortness of the night, but principally by his four moons or satellites.

10. His orbit is between those of Vesta and Saturn.

11. His satellites are four in number, some of which are nearly as large as the Earth.

12. The first is 262,000 miles distant from him, the second 423,000, the third 676,000, and the fourth 1,189,000.

13. They were discovered by Marius, mathematician to the Elector of Brandenburg.

14. The discovery is of great use, inasmuch as they enable us to determine the distance of Jupiter from us, to prove the progressive motion of light, and by their eclipses to find the longitude of places.

15. The nearest satellite performs its orbit round him in 4 days 3 hours ; the second in 8 days 12 hours ; the third in 17 days 12 hours ; and the fourth in 41 days.

16. When a satellite goes behind the planet it is said to be in occultation or hidden ; when it is in a line between Jupiter and the Earth, it is said to be making a transit over the disc.

17. His distance from the Sun is 495,000,000 miles.

QUESTIONS ON CHAPTER IX.

1. What appearance does Jupiter present?
2. What is his diameter and magnitude?
3. How long is he in moving round the Sun, and turning on his axis, and at what rate?
4. State what you know of his belts.
5. Why has he no difference of season?
6. What would be the consequence if his axis were much inclined?
7. What is the difference between his diameters?
8. How must the Sun appear to his inhabitants?
9. How is the deficiency of light supplied?
10. Where is his orbit placed?
11. How many satellites has he?
12. What are their distances from him?
13. By whom were they discovered?
14. Is their discovery of any use?
15. In what time does each perform its orbit?
16. When is a satellite in occultation, and when does it make a transit?
17. What is the distance of this planet from the Sun?

CHAPTER X.

SATURN (♄).

1. Before the discovery of Dr. Herschel's planet (which is called after his name), Saturn was supposed to be the most remote planet of the solar system.

2. He shines with a feeble light, and one would scarcely imagine such a luminous little body to be the most stupendous in size of all the planets.

3. His light is so feeble on account of his immense distance from the Sun and Earth.

4. His distance from the Sun is 900,000,000 miles, round which body he revolves in 30 years.

5. For many centuries it was supposed that Saturn had no diurnal rotation, but we now know that it is accomplished in 10 hours 6 minutes.

6. His diameter is by some said to be 67,000, by others 79,000 miles, or about ten times the diameter of the Earth.

7. On account of his immense distance from us, it is probable that many phenomena connected with him escape our notice; but one peculiarity is to be noticed, viz., that he is surrounded by two luminous rings, one outside the other, nearly as bright as himself, and sometimes even brighter.

8. We cannot say what are the nature and uses of these rings; they appear to be of the same nature as the planet, and, like him, are probably inhabited.

9. They only present luminous appearances when he is in certain signs of the zodiac.

10. When he is in the sign Sagittarius, and also when in Gemini, the rings appear quite distinct from his disc; but at other times they appear as a dark line on his body.

11. He has also belts like those of Jupiter, and is attended by seven satellites, which move round him nearly on the plane of the rings, and in certain positions they appear like roughnesses there.

12. The first is 170,000 miles distant from him; the second, 217,000; the third, 303,000; the fourth, 704,000; the fifth, 2,050,000; the sixth, 135,000; and the seventh, 107,000.

QUESTIONS ON CHAPTER X.

1. What was supposed before the discovery of Uranus?
2. What appearance does Saturn present?
3. Why is his light feeble?
4. What is his distance from the Sun? How long is he in performing his orbit?
5. How long is he in revolving on his axis?
6. What is his diameter?
7. What peculiarity is to be noticed about him?
8. Of what nature do the rings appear to be?
9. When do they appear luminous?
10. State farther what you know of their appearances.
11. Has he any belts or satellites?
12. Name their distances from him.

CHAPTER XI.

HERSCHEL (♅).

1. Herschel or Uranus is the most remote planet but one of the solar system. Its distance from the Sun is 1,800,000,000 miles, and it is distant from us 1,750,000,000 miles when nearest us.

2. Its diameter is about 35,000 miles, and its circumference 110,000, being 81 times larger than the Earth.

3. As this planet is 19 times farther from the Sun than the Earth, its light, derived from the former, must be 360 times less than ours.

4. Six satellites are connected with this planet, and the inclination of its orbit to the ecliptic is 46′ 26″.

5. If the degree of heat decreased in proportion to the distance from the Sun, it would follow that the amount of cold in this planet should be almost inconceivable ; but such is not the case ; the climate of Uranus, therefore, may be more genial, and there may be as much warmth as in the middle parts of our temperate zones.

6. Before it was known that this planet existed, some inequalities in the motions of Jupiter and Saturn were observed, which could not be accounted for from the mutual attractions of these planets ; it was therefore conceived that another might be beyond the orbit of Saturn, which produced these irregularities.

7. Sir W. Herschel, when living in Bath, had constructed more powerful and larger reflecting telescopes than any then in use. Whilst pursuing a design he had formed, viz., of searching every part of the heavens, he observed a star near the foot of Castor, which seemed to differ considerably from others. He at first supposed it to be a comet, but as no tail could be perceived, and as it showed a faint steady light, it was considered as an extraordinary comet. Lexel, a Russian astronomer, who was in London at the time, discovered its circular form, and therefore it was considered as a planet.

8. Another name by which this planet is known is the Georgium Sidus or George's star. Dr. Herschel proposed

that name in honour of the sovereign by whom he was so much patronized.

9. In 1846 another planet was discovered, which has been named Neptune, but from his immense distance, little or nothing is known of him.

QUESTIONS ON CHAPTER XI.

1. What is Herschel's distance from the Earth and Sun?
2. What is its diameter and circumference?
3. How much light does it derive from the Sun, in comparison with that of the Earth?
4. How many satellites are there, and what is the inclination of its orbit to the ecliptic?
5. What climate is Uranus supposed to have?
6. What led to its discovery?
7. Under what circumstances?
8. By what other name is this planet known? Why was it so called?
9. What planet forms the present boundary of the solar system?

CHAPTER XII.

THE FIXED STARS.

1. Those bodies which do not revolve in circular or elliptical orbits, but which remain always at the same distances from the Earth and from each other, are called fixed stars.

2. They are generally supposed to be suns, each being the centre of its own particular system, and having planets revolving about them.

3. Their distances from us are so great as not to be susceptible of calculation.

4. The parallax of a star is an angle at the body subtended by, or having for its base, the radius, or half the diameter, of the Earth.

5. A proof of the immense distances of these bodies is that they have no sensible parallax.

6. The diameter of the Earth's orbit is 190,000,000 miles, and therefore we are nearer to these bodies at some periods

than at others; yet their apparent magnitude is not increased. This is another proof of their immense distance.

7. Light travels at the rate of 12,000,000 miles per minute, and from the Sun to us in 8 minutes, *but yet* it is calculated that it would be one year and six months in travelling from the nearest fixed star to the Earth.

8. It is also computed that a cannon-ball, travelling at the rate of 20 miles per minute, would be 760,000 years in performing the same journey.

9. The number of these bodies cannot be determined. On a very clear night 1,000 may be counted without the aid of a telescope. With that instrument an infinite number may be discovered, and there is every reason to believe that the best instruments have never made visible the one-thousandth part of their number.

10. On account of their immense distance, they cannot receive any of the Sun's light; and as they do not move in orbits, it is more than probable that they themselves are suns.

11. They are not all of the same magnitude. Some are of the first; others of the second, third, or fourth; and the remainder are denominated nebulæ on account of their cloudy, indistinct appearance.

12. Those which appear to be the largest are not really so; it is probable that some which appear to be so small are really the largest, their great distance diminishing their magnitude.

13. They are divided into constellations, and are distinguished by the letters of the Greek alphabet.

14. The word constellation is from the Latin *con* (together), and *stellæ* (stars); it thus signifies a cluster or group of stars.

15. The following are the principal constellations of the northern hemisphere:—Ursa Major or the Great Bear, Ursa Minor or the Little Bear, Draco, Cepheus, Andromeda, Cassiopeia, Perseus, Pegasus, Canes Venatici, Bootes, Corona Borealis, Hercules, Lyra, Cygnus, Vulpecula, Aquila, Sagitta, Auriga, and Leo Minor.

16. The following are the principal of the southern hemisphere:—Cetus, Eridanus, Centaurus, Lupus, Triangulum Australe, Pavo, Corona Australis, Grus, Orion, Canis Major, Canis Minor, Piscis Australis, Lepus, Crux, Hydra, Corvus, Piscis Volans.

17. There are many remaining which are not formed into constellations; these are called "unformed stars."

18. The most remarkable of the nebulæ or cloudy stars is that in the centre of Orion's sword, the stars of which seem very lucid near the middle, but faint towards the edge.

QUESTIONS ON CHAPTER XII.

1. What are fixed stars?
2. What are they supposed to be?
3. What is known of their distances?
4. Define parallax.
5. What proves the immense distances of these bodies?
6. Give another proof.
7. How long is their light in travelling to us?
8. Prove their immense distance by the illustration of a cannon-ball.
9. What is known of their number?
10. What tends to prove that they are suns?
11. State what you know of their magnitude.
12. Do their appearances indicate their magnitudes?
13. How are they distinguished?
14. What is the meaning of constellation?
15. Name the northern constellations.
16. Name those of the southern hemisphere.
17. What are unformed stars?
18. Which is the most remarkable of the nebulæ?

CHAPTER XIII.

COMETS.

1. The word comet is from the Greek *comēs,* signifying hair.

2. They have the appearances of stars, some with a luminous train of the same colour as the body; others surrounded by luminous matter, the whole forming a circle of which the body itself is the centre.

3. The luminous point generally seen towards the centre is termed the *nucleus.*

4. The luminous train which follows some comets is called the *tail.*

5. These bodies were in ignorant times considered as subjects of terror, both on account of their being so seldom visible and also their extraordinary shapes, which were considered menacing.

6. The people regarded them as the presages of a great calamity, as frequently disasters either preceded or followed their appearances.

7. When the theories of Newton and Halley had become known, they ceased to be considered as accidental existences, and were looked upon as true celestial bodies following regular laws.

8. The comet of 1664 was considered by the vulgar as the forerunner of the deaths of all the sovereigns, but nevertheless none died during that year.

9. Some of these bodies frequently escape notice; when they traverse the sky during the day, they are seldom rendered visible, except on the rare event of an eclipse of the Sun.

10. According to Seneca, a comet which appeared 60 years before the birth of Christ was rendered visible by an eclipse of the Sun.

11. There have been some bright enough to be visible during the day, as, for instance, those of the years 44 B.C., 1402, and 1532.

12. Ordinarily they are only visible, when they are in a part of their orbit nearest the Sun: at that time their course is more accelerated.

13. The rate of travelling of Newton's comet was nearly 880,000 miles per hour: this body was at right angles to the Earth's orbit, and appeared in 1682.

14. The stars are not rendered invisible, when covered by the densest part of the comet, although a slight fog would be sufficient to make them so.

15. In 1705 Dr. Halley calculated the orbits of several comets, following the system of attraction formed by Newton. He discovered that the comet of 1531, 1607, and 1682 was the same which re-appeared in 1789. This comet appeared in 1835; the diameter of its orbit is 3,678,000,000 miles, and the interval between its appearances 75 years.

16. The periods of the appearances of a comet are not always regular; for instance Halley's comet appeared in 1759, and owing to the attraction of the neighbouring planets, it was kept back, when it should have re-appeared; this proved the theory of attraction.

17. The great comet of 1556, the return of which had been predicted for 1848, did not appear during that year; an astronomer of Middleburg, after an immense calculation, found that it would be retarded for two years; but this prediction was not fulfilled.

18. The comet of 1853 was at a distance of 80,100,000 miles from the Earth, it travelled at the rate of 450 miles per minute, or 648,000 per day, its diameter was nearly 8,000 miles, and its tail 4,500,000 miles in length.

19. It is generally agreed among astronomers, that the mass and density of comets are very small, and that their attraction cannot produce any sensible effect upon planetary bodies.

20. One appeared in 1770, which was again visible in 1778, and has not been seen since. It passed very near to Jupiter, and it has been supposed that the attraction of that planet has been the cause of its not re-appearing.

21. The comet discovered by Mr. Encke, and called after him, takes about 1,212 days to perform its orbit; this period is not constant, for from 1855 till 1858 it occupied 1,210 days.

22. Some others have been discovered by Donati and Brorsen, and called after them.

QUESTIONS ON CHAPTER XIII.

1. What is the meaning of the word comet?
2. Describe the appearances of some comets?
3. What is the nucleus?
4. What is the tail of a comet?
5. How were they regarded in ancient times?
6. On what account were they so regarded?
7. When did these suppositions cease to exist?
8. In what way was the comet of 1664 regarded?
9. Why are comets not always visible?
10. How was the comet of 63 B.C. rendered visible?
11. Mention some others which have been seen in the day time?
12. In what position are they ordinarily visible?
13. At what rate did Newton's comet travel? When did it appear?
14. Has a comet any effect on the appearances of the neighbouring stars?
15. What did Dr. Halley discover?

16. Do comets always appear at regular intervals?
17. What was calculated about the appearance of that of 1556?
18. State what you know of the comet of 1853?
19. Have comets any attraction on the planets?
20. What is known of the comet of 1770?
21. State what you know of Encke's comet?
22. By whom have others been discovered?

CHAPTER XIV.

THE AURORA BOREALIS.

1. This is a phenomenon which appears in the polar regions of the earth, from which fact it has been called the polar light. It was known to the ancients by the name of the burning torch.

2. The words Aurora Borealis are Latin, signifying northern dawn; the name has been given to it because when near the horizon it presents a similar appearance to the commencement of twilight.

3. It is rarely seen in this country, but is often visible in those bordering on the Arctic circle, as Lapland, Norway, and Siberia, where it relieves the monotony of the long nights; for the Sun does not appear, and therefore there is total darkness for about 6 weeks.

4. It may be seen as low as latitude 45°, and more frequently the nearer we approach the pole.

5. In the most northern parts it is seen during every season, and in every variety of form, often spread upon the horizon as a cloud or light smoke, having an arched form alternately more or less luminous, and of different colours.

6. It is most frequent at the time of the equinoxes; but no regular period has been assigned for its appearance.

7. After sunset a confused light may be seen towards the north, and soon after wide and diffused jets of light rise above the horizon.

8. After this, which is the prelude of the phenomenon, two vast fiery columns rise slowly one from the east and the other from the westward.

9. Whilst rising they change in colour and aspect, passing from yellow or deep green to a purple tint.

10. They then unite, and form one arch of an immense extent, and when formed it remains for hours ; the dark space towards the horizon is traversed by diffused and different coloured light which travels towards the zenith, and concentrates itself into a little circular space which is called the crown ; when this is formed the phenomenon is complete.

11. After some hours, the light gets weaker, the crown disappears, and soon after only fluctuating light can be perceived, which gradually vanishes.

12. The phenomenon is not confined to the polar regions ; it has been seen at St. Petersburg, Naples, Rome, Lisbon, Cadiz, and other places.

13. It has been estimated that it takes place, ordinarily, at a vertical altitude of between 300 and 900 miles.

14. It is rarely seen here during the day, but Arago states that one was seen in Perthshire at daylight.

15. Out of all the hypotheses proposed to explain the cause of this phenomenon, that most generally admitted is magnetism.

16. The Earth possesses the property of attraction ; consequently a bar of iron, nicely balanced and properly magnetized, if placed with its axis horizontal, will incline to the earth, and the angle it would make with a horizontal line is called the dip, which varies in different places : this is called a dipping needle.

17. If the same bar be placed with its axis vertical, so that the needle can move either to the right or left, and with its marked extremity or north pole to the north point of the horizon, it will move from that position, either towards the east or west : this is the principle of the mariner's compass, and its deviation from the true north is called its variation, which changes according to the place.

18. The point to which the marked end of the needle is directed, is called the magnetic north ; the opposite point therefore will be the magnetic south.

19. The summit of the arch of an Aurora Borealis, is always upon the magnetic meridian.

20. This phenomenon ordinarily changes the magnetic needle from its position, both as regards dip and variation, and even in places where it is not visible.

21. Arago has noticed that as soon as it is visible in the polar region, it changes the direction of the needle at Paris; if visible in the morning it changes the needle to the west, and if in the evening, to the east. Thus the change of direction of the needle will indicate whether the phenomenon is visible at Lapland, or Greenland, or at any other part of the polar regions.

22. This was the case on March 29th 1826, when Arago observed unaccountable changes in the needle, and at the same time Mr. Dalton observed a luminosity at Manchester, in the north quarter of the heavens.

23. It has also been observed to affect the electric telegraph. Mr. Highton states that a wire which passed under Nalford tunnel, was unserviceable for three hours. In one case he traced the effect from Northampton towards Shapstone and Peterborough, upon the route of the eastern telegraphic wire to London.

24. The Aurora Borealis sometimes descends so low, that its effect upon travellers who ascend high mountains is analogous to that of wind.

25. There is a similar phenomenon in the southern hemisphere, called the Aurora Australis, which has been observed by Captain Cook, and other navigators who have visited that part of the globe.

26. The Aurora Borealis has been noticed by the ancients. Pliny gives the account as follows:—"We have seen torches, burning lamps, lances, and pieces of soft wood, a fire which seems to fall upon the earth, and rain blood." "We have seen armies in the sky, they appear to touch each other, and we have heard the noise of arms, and the sound of trumpets."

27. Towards the end of the sixteenth century, (the time of the appearance of this phenomenon in France), 10,000 or 12,000 penitents came to the Cathedral of Notre Dame to offer prayers, having seen sights in the sky.

28. The chronicles of the same age speak of the appearance of blood-stained arms in the sky, as the presage of great misfortunes. Gassendi saw the same with the eye of a philosopher, and in September 1624 gave it the name of Aurora Borealis.

QUESTIONS ON CHAPTER XIV.

1. Why has this phenomenon been called the polar light? Under what name was it known to the ancients?
2. What is the meaning of Aurora Borealis: why has it been so called?
3. Where is it mostly visible?
4. What is the least latitude in which it has been seen?
5. Describe its appearance.
6. When is it most frequently seen?
7. What is the prelude to its appearance?

8, 9. Describe the appearance farther.

10. How are the arch and crown formed?
11. How does it disappear?
12. Where has it been seen?
13. At what height does it generally happen?
14. Give an instance of its appearance during the day.
15. To what is it generally ascribed?
16. Describe the dipping needle.
17. What is meant by the variation of the compass?
18. To what part is the marked end of the needle directed?
19. Where is the summit of the arch placed?
20. How does the phenomenon affect the needle?
21. Give an instance.
22. Give another instance.
23. Give an instance of its effect upon the electric telegraph.
24. What effect has it upon travellers?
25. What is the Aurora Australis?
26. Give Pliny's account of the Aurora Borealis.
27. What effect had it upon the ancients?
28. What is said of it in the chronicles of the sixteenth century?

CHAPTER XV.

METEORS.

1. Bodies which seem composed of flame, and which move with an extreme rapidity, are commonly known by the name of shooting stars, and during their course in the sky they leave behind them a brilliant train.

2. It often happens that they disappear without any farther phenomenon, but sometimes they are accompanied with detonations or explosive sounds, resembling those of cannon, and end in the fall of a projectile.

3. The stone which falls is composed of silica or flint, iron, and nickel ; the two latter are not in the same state as when found in the earth.

4. They have been known in the ancient times : a certain philosopher said they fell from the sun, supposing that body to be one immense meteor.

5. At the time when this theory was adopted an immense stone fell in Thrace. This was the first phenomenon of the kind mentioned by historians.

6. Projectiles of the same kind fell in the town of Cassandria in Macedonia.

7. The following hypotheses have been proposed to explain these curious phenomena :—1. That they were formed in the atmosphere, and were of the same nature as hailstones. 2. Laplace thinks that they draw their origin from volcanoes in the moon. 3. Others think that they were fragments of a planet which moving in space have come into the earth's atmosphere, and at last fallen.

8. It is generally decided that they are not projected from volcanoes in the moon ; if it were so, they would not contain the precise number of ingredients always found in them.

9. The third supposition was made by Sir David Brewster ; it is, perhaps, the most reasonable, inasmuch as it would account for the heavy substances which fall.

10. The velocity of these bodies is nearly 36 miles per second, or about double that with which the earth moves round the sun.

11. On the night of the 15th November 1857, several persons at Beaune, in Burgundy, observed a luminous meteor, which changed greatly before it disappeared. There was at first a long wide train of light, having the appearance of a column of flame ; it fell obliquely from the south-east to the south towards the horizon. In descending, it formed an immense globe of fire, which was at last extinguished. The spectators experienced a feeling as if the ground had been shaken under their feet, and a disagreeable odour, as of sulphur, arose when the light was extinguished.

12. Another was seen on January 5th, 1856, about 5 p.m., both at Havre and Rouen, which had the appearance of a ball of fire, throwing out a luminous train, which did not disappear

until 20 minutes after the body. This was visible also in England.

13. A third was seen on February 3rd, about 8 p.m.; its direction was north-east, and it was of a blue colour.

14. Another was seen at Hartlepool in December 1855, about 3 p.m., when a strong north-west wind arose, bringing hail and rain, and a thick cloud obscured the town for several minutes. When this disappeared, the inhabitants were surprised at seeing a large ball of fire, apparently at a very little altitude above the town. After 15 seconds a sound was heard similar to the discharge of guns, sulphureous vapours arose, and many of the people experienced a kind of sensation like that produced by electricity.

QUESTIONS ON CHAPTER XV.

1. What are meteors?
2. With what are they sometimes accompanied?
3. Of what is the stone composed?
4. What was said of them in ancient times?
5. Have they been mentioned by historians?
6. Where else have stones fallen?
7. What hypotheses have been proposed as explanations?
8. What proves that they do not come from the moon?
9. Why is Sir D. Brewster's supposition the most reasonable?
10. At what rate do meteors travel?
11. Describe the meteor of November 1857.
12. Describe that of January 1856.
13. When was there another seen: what was its direction?
14. State what you know of that which appeared at Hartlepool.

CHAPTER XVI.

THE ZODIACAL LIGHT AND MILKY WAY.

1. The Zodiacal Light is a phenomenon generally considered as connected with the Sun.

2. It is supposed to have been first observed by Mr. Childrey, in 1660, but was more particularly noticed in 1683.

3. It appears in a conical form, having its base directed to the body of the Sun, and its apex or point towards some one star in the constellations of the Zodiac.

4. Its light presents the same appearance as that of the Milky Way, and is thin enough for the stars to be seen through it.

5. It is weaker in the morning than at night, and disappears in moonlight or twilight.

6. In north latitudes it is most conspicuous after evening twilight in the month of March, and before morning twilight in October.

7. Humboldt, the celebrated traveller, saw it at Caraccas, on the 18th of January, at seven o'clock in the evening.

8. It is generally ascribed to an atmosphere of immense extent surrounding the Sun, and extending to the orbit of Mercury.

9. Some identify this phenomenon with the cause that produces meteors.

10. On examining the sky at night we discover a pale irregular light, forming a band or belt, which extends across the heavens from S.S.E. to N.N.W ; this is called the Milky Way.

11. It resembles a light cloud, and is composed of an innumerable quantity of stars, none of which can be distinguished without the aid of a telescope.

12. On account of their immense distance, they cannot be seen with the naked eye, and even with the telescope we see spaces to all appearance occupied by other stars which that instrument cannot render visible.

13. Some of these, it is supposed, are larger than the Earth, yet they appear, even through the telescope, as mere points ; which fact shows their immense distance.

QUESTIONS ON CHAPTER XVI.

1. What is the Zodiacal Light ?
2. By whom was it first observed, and when ?
3. Of what form is it, and how is it placed ?
4. What appearance does it present ?
5. When is it weakest ?
6. When is it most conspicuous in north latitudes ?

7. By what traveller has it been observed, and where ?
8. To what is it ascribed ?
9. To what cause do some ascribe it ?
10. What is the Milky Way ?
11. What is its appearance ?
12. What proves that it is probably composed of stars ?
13. How is their immense distance proved ?

CHAPTER XVII.

THEORY OF ECLIPSES.

1. The first explanation of these phenomena given by the ancients was entirely theological. They pretended that the deities, tired of the amount of crimes committed on the earth, sent them as a punishment ; this idea caused them to be considered as the presages of disasters.

2. Pericles was the only ancient philosopher who knew the true causes of eclipses, and he could not refrain from showing others the absurdity of their suppositions.

3. Agathocles, on his arrival in Africa, was surprised by an eclipse of the Sun, and his soldiers refused to march against the Carthaginians : but when he persuaded them that it was an omen of misfortune to their enemies, he marched against them and was victorious.

4. The Sun is a luminous body, and consequently he can only be eclipsed by the interposition of a dark body between him and us. The Moon, on the contrary, being dark, and receiving her light from the Sun, it is sufficient for an opaque body to pass between her and the Sun, in order that she may be eclipsed : the body which does this is the Earth.

5. When a large body is eclipsed by a smaller one, a shadow is cast which is of a conical or sugar-loaf form ; therefore an eclipse of the Sun is visible only to that part of the Earth where the shadow falls, and it is rendered visible to greater tracts according as it falls beyond the surface ; that is, all within the shadow will see the eclipse.

6. Eclipses are either total or partial ; they are total when the whole of the eclipsed body is covered, and partial when only a part is covered.

7. An eclipse of the Sun is caused by the interposition of the Moon between the Earth and that body.

8. If the eclipse be total, then the apex or point of the shadow must fall behind the Earth, and it is visible to those parts of the Earth within the shadow.

9. If it be partial, the cone or shadow does not reach the Earth.

10. During a total eclipse the Sun is not entirely hidden for more than five minutes.

11. The effects of the obscurity during an eclipse are seen in animals. In that of 1842 the dogs howled, the birds suspended their flight, and all things presented the same appearance as if the Sun had sunk beneath the horizon.

12. Arago gives a remarkable instance of the effect of eclipses of the Sun on animals. A dog had been deprived of nourishment for a certain time, and just before the commencement of the eclipse a piece of bread was given to him, which he commenced eating, but at the moment the Sun disappeared he stopped, and did not recommence until the termination of the phenomenon.

13. When the Sun's light is entirely lost, stars of the first magnitude may be seen, but their light is weak, owing to the reflection of the rays of light.

14. The most remarkable phenomenon connected with the eclipse of July 1842, was the appearance of two red protuberances, which were beyond the Moon's disc, always retaining the same colour, but presenting varied forms.

15. This engaged the attention of astronomers; some believed them to be the reflection of light upon certain mountains of the moon, whilst others considered them as luminous clouds detached from the Sun's atmosphere.

16. There is said to be an annular eclipse of the Sun when it is not entirely covered, a ring being left by the Sun's edge being visible. The word annular is from the Latin *annulus*, (a ring).

17. During the solar eclipses of July 1851, and December 1852 (the former visible in the north of Europe, and the latter in China and Japan), similar protuberances to those already mentioned were seen; and it has been since agreed that they were luminous clouds detached from the Sun's atmosphere.

18. A total eclipse occurred on July 18th, 1860, visible in the north of America, Spain, and the north of Africa, and as a partial one in England.

19. It is calculated that others will happen as follow:—22nd December 1870, in the Azores, Spain, Algeria, Sicily, and Turkey; 19th August, 1887, in the north-east of Germany, Russia, and Central Asia; 9th August, 1896, in Greenland and Siberia; 28th May, 1900, in the United States, Spain, Algeria, and Egypt.

20. The Moon being an opaque body receiving her light from the Sun, an eclipse of her must be caused by the Sun's light being intercepted by some other planet, viz., the Earth.

21. It does not follow that she should be eclipsed whenever we are between her and the Sun. (See Chap. VI. § 31, 32.)

22. To illustrate the manner in which she is eclipsed, place a ball so that the light of a lamp may fall on it, the latter will represent the Sun, and the former the Moon, and as the side next the lamp will be enlightened, the same appearance will be presented as that of the Moon at the full. If another ball be brought between (representing the Earth) so as to intercept the light of the lamp, the first ball will be eclipsed.

23. When the Moon is eclipsed she must be at a very little distance from the base of the shadow cast by the Earth, or in her nodes (See Chap. VI. § 34).

24. The duration of a total eclipse is two hours, and the total duration of the phenomenon four hours; that is, the Moon is entirely hidden during the former time, and the whole phenomenon lasts during the latter.

25. In a partial eclipse the Moon is in such a position that the Earth casts only a part of its shadow on her. The length of the phenomenon depends on how much the Moon is covered by the shadow, thus on October 13th, 1856, 49 parts out of 100 in her diameter were eclipsed.

26. On looking at the disc of the Moon we perceive that it has a reddish tint, which is caused by the terrestrial atmosphere.

27. The rays of the Sun which penetrate our atmosphere fall afterwards into the interior of the shadow, and have been bent by the former. At the moment of the eclipse, the Moon receives many of the rays, as if she were not eclipsed, but

these, having had their blue colour absorbed by the air, have a red sombre tint.

28. The interposition of the terrestrial atmosphere would render the Sun's appearance very curious to an observer in the Moon. When we have an eclipse of that body, her inhabitants would have an eclipse of the Sun.

29. The Sun, as seen from the Moon, must have an apparent diameter of half a degree, and the Earth two degrees; therefore we should suppose he would be totally eclipsed; but it is not so, for the Earth's atmosphere would cause the Sun to appear larger than he really is: it is therefore probable that he is not totally eclipsed, excepting when the Earth is surrounded by clouds, and completely covers him.

30. Although the sky may be covered with clouds at the time of an eclipse of the Moon, we are struck with the unusual obscurity.

31. When the eclipse is at its height, or the darkness greatest, the stars appear particularly bright.

32. Eclipses of the Moon are visible to all those places where she is above the horizon, but the hour is not the same everywhere; it depends on the longitude, for the Moon comes to a place east of Greenwich before one west. The difference between the time of the eclipse at the place and the time at Greenwich will give the longitude of the place in hours.

QUESTIONS ON CHAPTER XVII.

1. What was the ancient explanation of eclipses?
2. Was there any philosopher who knew their cause?
3. Give an instance of their effect on the ancients.
4. Show how the Sun and Moon are eclipsed.
5. To what parts of the Earth is an eclipse visible?
6. When are eclipses total? When partial?
7. How is an eclipse of the Sun caused?
8. What is the consequence if the eclipse be total?
9. What follows if it be partial?
10. For how long is the Sun hidden during a total eclipse?
11. Give an instance of its effect on animals.
12. What instance does Arago give?
13. What occurs when the Sun's light is lost?
14. What appeared during the eclipse of 1842?
15. What explanation was given of this?
16. When does an annular eclipse happen? What is the meaning of annular?

17. What appeared during the eclipses of 1851 and 1852 ?
18. Where was that of 1860 visible ?
19. When will others happen, and where ?
20. What causes an eclipse of the Moon ?
21. Why does it not follow that she should be eclipsed when between the Sun and us ?
22. Illustrate the way in which she is eclipsed.
23. At what part of the Earth's shadow must she be ?
24. How long does the phenomenon last, and during what time is she hidden ?
25. On what does the duration of the phenomenon depend ?
26. What do we perceive on looking at the Moon ?
27. Explain the cause of this.
28. What phenomenon would the inhabitants of the Moon have ?
29. Why may not the Sun be totally eclipsed to them ?
30. What effect is always seen from an eclipse of the Moon ?
31. What may be noticed about the stars ?
32. To what places are the eclipses visible ?

CHAPTER XVIII.

THEORY OF THE TIDES.

1. The tides are alternate movements of the ocean, owing to the attraction of the Sun and Moon on the waters.

2. In the space of 24 hours 50 minutes a great wave is carried from the equator to the poles, and from the poles to the equator. This, in its passage over the earth, causes the water to rush up the rivers, and each place to which it comes has what is called high water.

3. When the wave has passed, the water gets lower and lower until it is low water.

4. The high tides take place when the water is going from the equator towards the poles, and low water when it returns.

5. The first of the Greeks who noticed the cause of the tides was Pytheas, who lived 300 years before Christ.

6. The Moon passes successively in front, as it were, of each point of the ocean, and, by virtue of the law of attraction, it draws the waters into a heap. This heap moves as already stated, and is called a tidal wave.

7. On account of the irregular motion of the Moon, she comes to the meridian about 50 minutes later every day; it is then high water at the place; and it follows that the time of high water is later every day by the same amount.

8. The tides are much higher when the Moon is in perihelion, or nearest the Earth.

9. At full or new moon the tides are greatest, for the Sun joins its attraction to that of the Moon; but at the time of her quadrature they are least.

10. The waters on that side of the Earth which is turned away from the Moon are less attracted than the centre of the Earth, or the Earth is drawn away from the water; consequently another tidal wave is formed in the opposite hemisphere. We have, therefore, two tidal waves in 24 hours 50 minutes.

11. The attractive force of the Sun on the Earth, although not above one third of that of the Moon, is sufficient to produce a flux and reflux; hence two sorts of tides may be distinguished, viz., solar and lunar.

12. The most elevated part of the tidal wave is not exactly under the Moon, but always a little to the east, though never more than 15 degrees.

13. This is because the waters do not immediately obey the attraction of the Moon, on account of the resistance they meet with in the forms of the land.

14. Newton has calculated that if there were tides in the Moon the attraction of the Earth would be sufficient to raise the waters to seven times the greatest height of our tidal waves.

QUESTIONS ON CHAPTER XVIII.

1. What are tides? How are they caused?
2. How is high water caused?
3. What is the cause of low water?
4. When do high tides take place?
5. What ancient noticed their cause?
6. What is a tidal wave? How is it caused?
7. Why is the time of high water 50 minutes later every day?
8. What increases the height of the tides?
9. When are they greatest?

10. Why have we two tidal waves? In what time?
11. What influence has the Sun on the waters?
12. Where is the most elevated part of a tidal wave?
13. How is it retarded?
14. What calculation has Newton made?

CHAPTER XIX.

CIRCLES OF THE EARTH.

1. A circle is a figure contained or bounded by one line called the circumference, and all lines drawn from its centre to the circumference are equal, and are called radii. One of these is called a radius.

2. The principal circles on the surface of the Earth are, the equator, meridians, and parallels of latitude.

3. Circles are divided into two classes, great and less circles.

4. A great circle is one whose plane (or imaginary flat surface, bounded by the circumference,) passes through the centre of the sphere on which it is drawn.

5. A less circle is one whose plane does not pass through the centre of the sphere.

6. A sphere is a solid body all the diameters of which are equal. If it be cut straight through in any direction the flat surface exposed will be a circle.

7. The earth is not a perfect sphere, for it is flattened at the poles, one diameter being longer than the other by 26 miles.

8. A meridian is a great circle, passing through both poles, and cutting the equator perpendicularly or at right angles.

9. The equator is a great circle passing round the Earth, at an equal distance from each pole.

10. Parallels of latitude are less circles, passing round the Earth, parallel to, or running in the same direction with, the equator.

11. The latitude of a place is its distance north or south of the equator, measured on a meridian, or the arc or part of a meridian, intercepted between the place and the equator.

12. The longitude of a place is its distance east or west of some particular meridian measured on the equator. The meridian from which it is measured is called the first meridian.

13. Englishmen measure their longitude from the meridian of Greenwich; Frenchmen from that of Paris.

14. Latitude and longitude are measured in degrees, marked thus (°).

15. A degree is equal to 60 geographical miles, or about 69½ English or common miles. Each geographical mile is called a minute, marked thus (′), and minutes are again divided into 60 parts called seconds (″).

16. Thus the latitude of Greenwich is 51° 28′ 50″, and its longitude is 0°.

17. The difference of latitude between two places is the arc of a meridian intercepted between them.

18. The difference of longitude between two places is the arc of the equator intercepted between the points where the meridians cut that circle.

19. When the places are on different sides of the equator their difference of latitude is the sum of their latitudes, and when they are on different sides of the first meridian their difference of longitude is the sum of their longitudes.

20. The greatest latitude a place can have is 90°, or from the equator to one of the poles; the greatest longitude is 180°, or half round the Earth.

21. A circle is divided into 360°; therefore half a circle is 180°, and a quarter of it is 90°.

22. The Earth being nearly a sphere, it is evident that the equator must be greater than any circle drawn parallel to it.

23. Hence, if the longitude of a place were measured on a parallel of latitude passing through it, it would not be the same as if measured on the equator, but the length of a degree would be diminished the nearer we approached the poles: for this reason a degree at the equator is taken as the measure of longitude.

24. It has been already stated in Chapter IV. (§ 13) that the Earth moves round the Sun, with its axis pointing in the same direction; this causes the Sun to appear to describe in the heavens throughout the year a circle, which appears to

cut the equator at an angle of 23½°; this circle is called the ecliptic.

25. On maps of the world this circle is represented by a curved line, which rises as much above the equator into the northern hemisphere, as it goes below into the southern, cutting the equator in two opposite points.

26. The surface of the Earth is divided into five zones or belts: viz., two frigid or cold, extending from each pole to the borders of the Arctic and Antarctic circles respectively; one torrid or hot, extending 23½° on each side of the equator; and two remaining parts called temperate zones.

27. There are two particular parallels of latitude called the tropics of Cancer and Capricorn, the former passing through the most northern point and the latter through the most southern point of the ecliptic. These mark the boundaries of the torrid zone on each side of the equator.

28. On the 21st of March the Sun is over one of the points where the ecliptic cuts the equator, on the 21st of September he is above the other; on the 21st of June he is over the tropic of Cancer, and on the 21st of December over that of Capricorn.

29. If the Earth's axis were perpendicular to the plane of its orbit, the parts round the poles would receive little or no light from the Sun; if parallel, then the parts at the equator would be in a similar position, and one pole would receive no light.

QUESTIONS ON CHAPTER XIX.

1. What is a circle?
2. Name the principal circles on the Earth.
3. Into what are they divided?
4. What is a great circle?
5. Define a less circle?
6. What is a sphere?
7. Of what shape is the Earth?
8. What is a meridian?
9. What is the equator?
10. Define parallels of latitude.
11. What is the latitude of a place?
12. What is its longitude?
13. From what places do Englishmen and Frenchmen measure?
14. How are latitude and longitude measured?
15. How is a degree divided?

16. What is the latitude and longitude of Greenwich ?
17. Define difference of latitude.
18. What is difference of longitude ?
19. To what is each equal, when the places are in different hemispheres and on opposite sides of the first meridian ?
20. What is the greatest latitude and longitude a place can have ?
21. How is a circle divided ?
22. What follows from the spherical shape of the Earth ?
23. Why is longitude measured on the equator ?
24. What is the ecliptic ?
25. How is it laid down in maps ?
26. How many zones or belts are there ?
27. What are the tropics of Cancer and Capricorn ?
28. When is the Sun at each ?
29. What would be the consequence if the Earth's axis were perpendicular to the plane of its orbit ? what if parallel ?

CHAPTER XX.

CIRCLES OF THE HEAVENS.

1. If we conceive a meridian on the Earth's surface to be produced, or its plane to be spread out to the heavens, we have what is called the celestial meridian.

2. In like manner if the plane of the equator be produced, we have the celestial equator.

3. As a meridian on the earth's surface is perpendicular to the equator, so is the celestial meridian perpendicular to the celestial equator.

4. The ecliptic is the Sun's imaginary path among the fixed stars, and as these bodies are for the most part formed into constellations, it will be seen that the Sun only moves amongst certain of them.

5. The ecliptic is divided into twelve parts, called signs of the Zodiac, which are—Aries (the ram), Taurus (the bull), Gemini (the twins), Cancer (the crab), Leo (the lion), Virgo (the virgin), Libra (the balance), Scorpio (the scorpion), Sagittarius (the archer), Capricornus (the goat), Aquarius (the water-bearer), Pisces (the fishes).

6. As the Sun is one year in performing his course in the ecliptic, it follows that he is in each of these signs during the space of one month.

7. From the 21st of March till the 21st of April he is in Aries, from the 21st of April till the 21st of May in Taurus, and so on, entering each sign on or about the 21st of each month, till on the 21st of February he is in Pisces.

8. Aries being the first point on the ecliptic, we reckon from it.

9. The longitude of a heavenly body, is its distance east or west of the first point of Aries, measured on the ecliptic.

10. The latitude of a heavenly body, is its distance above or below the ecliptic, measured on a meridian.

11. An angle on plane surfaces is the inclination of two straight lines; on a sphere it is made by that of two circles, and is called a spherical angle.

12. A spherical angle included between two meridians, one passing over the first point of Aries and the other over a heavenly body, is called the Right Ascension of the body, and is measured at the pole of the heavens.

13. Circles drawn parallel to the celestial equator are called circles of declination, and correspond to parallels of latitude on the Earth's surface.

14. The declination of a heavenly body is its height above the celestial equator, measured on a meridian.

15. If we conceive the Earth's axis to be produced to the celestial meridian, we get the north and south poles of the heavens.

16. The point in the heavens directly over a person's head is called the Zenith, the corresponding point below is called the Nadir.

17. The plane on which an observer on the Earth stands, produced to meet the celestial meridian, is called the sensible horizon.

18. Another one parallel to it, passing through the centre of the Earth, is called the rational horizon.

19. We have then these points on the celestial meridian,—the north point of the horizon, the elevated pole (of the same name as the latitude), the zenith, the elevated part of the equator, the south point of the horizon, the depressed pole, the nadir, and the depressed part of the equator.

20. An angle at the pole, included between a meridian passing over a body, and the celestial meridian, is called the hour angle.

21. This angle, in the case of the Sun, vanishes at noon, when he comes on the meridian, and is greatest just before noon.

22. An angle at the zenith included between a meridian passing over a body, and the celestial meridian, is called the azimuth of the body.

23. We say generally that the Sun rises in the east and sets in the west, but this is not strictly true, for he only does so on the 21st of March and the 21st of September.

24. On the 21st of March he rises in the east point, and travels for that day on the equator, and sets in the west point.

25. Between the 21st of March and that of June he rises more northerly, and consequently appears earlier; he then travels on a parallel of north declination, and sets later; therefore the night is shorter.

26. Between the 21st of September and that of December he rises more towards the south, and travels on a parallel of south declination; he therefore rises later and sets earlier, and the night is longer.

27. The distance of his rising point from the east, is called his rising amplitude; it increases from March to June, and decreases from June to September, being reckoned from east towards north.

28. Between September and December it increases from east towards south, and from December to March it decreases.

29. The amplitude of a body may also be defined, as an angle at the zenith, made by two circles, one passing over the object, and the other over the east point when rising, and the west when setting.

30. A circle passing through the zenith, nadir, and the east and west points of the horizon, is called the prime vertical circle.

31. Another passing through the poles and the east and west points, is called the six o'clock hour circle, because the Sun is on it, both in the morning and evening at that hour.

32. The declination of the Sun (*see* § 14.) increases from March till June northerly, and decreases from June till September, from which time till December it increases southerly, and again decreases from December till March.

33. The greater the declination of a body, the higher above the equator is the parallel on which it travels.

34. The Sun never attains a greater declination than 23½° north or south.

35. Circles of altitude are great circles passing through the zenith, and perpendicular to the horizon.

36. The altitude of a heavenly body is its height above the horizon, measured on a circle of altitude.

37. The greatest altitude a celestial body can have, is 90°; it is then in the zenith.

38. The greatest altitude of the Sun during the day, is at 12 o'clock or noon; his greatest altitude during the year, is at 12 o'clock on the 21st June.

39. The altitude subtracted from 90° gives his zenith distance, which must be always less than 90°.

QUESTIONS ON CHAPTER 20.

1. What is the celestial meridian?
2. Define the celestial equator.
3. What is the position of the celestial meridian with respect to the equator?
4. How may the ecliptic be defined?
5. Name the signs of the zodiac.
6. How long does the Sun continue in each?
7. When does he enter each?
8. From what point do we reckon?
9. What is the longitude of a body?
10. Define its latitude.
11. What is a spherical angle?
12. What is the right ascension of a body?
13. What are circles of declination?
14. Define declination.
15. What is meant by the poles of the heavens?
16. Define the zenith and nadir.
17. What is the sensible horizon?
18. What is the rational horizon?
19. Name the points on the celestial meridian.
20. What is meant by the term hour angle?
21. How does it increase and diminish?
22. Define azimuth.
23. When does the Sun rise in the east and set in the west?
24. Trace his course on the 21st of March and September.
25. When has he north declination?
26. When has he south declination?
27. What is meant by his rising amplitude.
28. How does his amplitude increase and decrease?
29. Give another definition of amplitude.
30. What is the prime vertical circle?
31. What is the six o'clock hour circle? Why is it so called?

32. How does the declination increase and decrease?
33. On what does the height of a body depend?
34. What is the greatest declination of the Sun?
35. Define circles of altitude.
36. What is meant by altitude?
37. What is the greatest altitude a body can have?
38. When is the Sun at his greatest altitude throughout the day and year?
39. How is his zenith distance found?

CHAPTER XXI.

DIVISIONS OF TIME.

1. The rotation of the Earth on its axis, causes each meridian to be brought under the Sun, or causes the Sun to appear to come on the celestial meridian.

2. When a body comes to the meridian of a place it is said to make a transit, as, for instance, the Sun comes every day at 12 o'clock to the meridian of the place, which is called the transit of the Sun for that day.

3. If we consider the Earth as stationary, then the celestial meridian will be stationary, and the heavenly bodies will appear to describe circles in the heavens, and come to it in succession.

4. As the Sun's motion does not afford an accurate measure of time, we have an imaginary or mean sun, which travels on the equator at an uniform rate, and is therefore sometimes before and sometimes after the true sun.

5. The interval between two transits of the mean sun, is called a mean solar day; it is the time by which we regulate our clocks and watches, and is exactly 24 hours long.

6. A sidereal day is the interval between the transits of the first point of Aries.

7. A lunar day is the interval between the transits of the moon.

8. An angle at the pole included between two meridians, one passing over the true sun and the other over the mean sun, is called the equation of time.

9. Hence it follows that in order to get the true time the equation of time must be added or subtracted according as the true sun is in advance of the mean sun.

10. On the 21st of March and September, the mean and true suns travel on the equator; they come to the meridian together, therefore the equation of time vanishes.

11. On the 21st of June and December it is greatest.

12. The commencement of an astronomical day is marked by the transit of the mean sun.

13. Thus, if on March the 1st, the sun come on the meridian at 12 o'clock, it is astronomically expressed as March 1st, 0 hours, and March 2nd does not begin till the following day at the same time.

14. Our ordinary mode of reckoning time by the clock is called civil time, A.M. or P.M., but 12 at noon is 0 hours astronomically, 2 A.M. is 14 hours, but 2 P.M. is the same, namely, 2 hours.

15. In addition to the separate motions of each body in the heavens, it has been proved that the whole mass of bodies, as it were, is moving onwards, and therefore with it, the first point of Aries; this theory is called the precession of the equinoxes.

16. In consequence of this, our seasons will be advanced every year, although on account of the small difference, it may hardly be perceived. Our pole star, (at present (*a*) Ursæ Minoris,) will be changed for another, but this cannot take place for a great length of time.

17. The hour angle of the mean sun measures the time, thus when we say it is 3 P.M., the angle is 3 hours west, but when 10 A.M., we say 22 hours; therefore we reckon the angle as increasing, till the sun comes to the meridian on the next day, when it is again 0 hours.

18. From this it appears that in order to get the astronomical time from civil A.M. time, we must add 12 hours, and reckon from the preceding day.

19. To obtain the angle in degrees we must multiply by 15, for the sun passes over 15° of the earth's surface in one hour, therefore 15° are equal to one hour.

20. In order to get the number of hours, from a certain number of degrees, we must divide the degrees by 15.

QUESTIONS ON CHAPTER XXI.

1. What causes the sun to appear to come to the meridian of a place ?
2. When does a body make a transit ?
3. Why do the heavenly bodies appear to describe circles?
4. What is the mean sun ?
5. Define a mean solar day.
6. What is a sidereal day ?
7. What is a lunar day ?
8. Define the equation of time.
9. How is the true time found?
10. When does the equation of time vanish ?
11. When is it greatest ?
12. When does an astronomical day commence?
13. Give an example of civil date expressed astronomically.
14. Show how it is so expressed.
15. What is the precession of the equinoxes ?
16. What effect will it have ?
17. How is the hour angle reckoned ?
18. How may civil date be converted into astronomical date ?
19. Why do we multiply by 15 to get the hour angle in degrees ?
20. How do we obtain hours from degrees ?

PART II.

TRIGONOMETRY AND MATHEMATICAL ASTRONOMY.

PART II.

TRIGONOMETRY AND MATHEMATICAL ASTRONOMY.

PLANE TRIGONOMETRY.

CHAPTER I.

ELEMENTARY FORMULÆ.

Plane Trigonometry shows us how to compute certain parts of a plane triangle, other parts being given. As the sides and angles are of different kinds with respect to measurement, they cannot be compared with each other, but the relation existing between the sides and angles, and the sizes of the angles, may be discovered by comparing the sides with certain lines in a circle, on which lines the arcs of the circle which measure the angles depend.

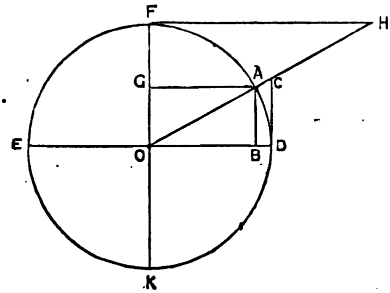
If the circumference of any circle be divided into 360 equal parts, each part is called a degree; the 60th part of a degree is a minute, and the 60th of a minute is a second; and whatever number of degrees, minutes, and seconds are contained in any angle, the arc which measures it contains the same.

Two arcs whose sum is equal to a semicircle, or two angles whose sum is equal to two right angles, are called *supplements* of each other.

Two arcs whose sum is a quadrant, or two angles whose sum is equal to a right angle, are called *complements* of each other.

Let FEKD be a circle; ED its diameter, OF a perpendicular, which, if produced to K, gives a diameter FK at right angles to ED.

Fig. 1.



Take any point A in the arc FD. Draw AB perpendicular to OD and AG to OF. From D draw DC a tangent to the circle at D, and FH a tangent at F. Join OA, and produce it to meet FH in H.

Then AB is called the sine of the arc AD or angle AOB.

OB	”	cosine	”	”
DC	”	tangent	”	”
FH	”	cotangent	”	”
OC	”	secant	”	”
OH	”	cosecant	”	”
BD	”	versed-sine	”	”
EB	”	suversed-sine	”	”

From the above definitions, the following is evident :—

1st. When an arc vanishes or becomes nothing, its sine, tangent, and versed-sine become nothing; its secant and cosine are equal to the radius; and its cotangent and cosecant are infinite.

2nd. The sine and versed-sine of a quadrant are each equal to the radius, the cosine is nothing, and the secant and tangent are infinite.

If we suppose the radius to be unity or 1, then from the right-angled triangle ABO we get the following formulæ, supposing, for convenience, the angle AOB or the arc AD to be A :—

In the right-angled triangle ABO

$$AO^2 = AB^2 + BO^2; \qquad (47, 1\text{ Eu.})$$

$$\text{but } AO^2 = \text{rad}^2 = 1,$$

$$AB^2 = \sin^2 A,$$

$$BO^2 = \cos^2 A.$$

(1) $\therefore 1 = \sin^2 A + \cos^2 A,$

whence (2) $\text{Sin}^2 A = 1 - \cos^2 A;$

and (3) $\text{Sin } A = \sqrt{1 - \cos^2 A},$

also (4) $\text{Cos}^2 A = 1 - \sin^2 A;$

and (5) $\text{Cos } A = \sqrt{1 - \sin^2 A}.$

Again, in the right-angled triangle OCD,

$$OC^2 = OD^2 + DC^2.$$

(6) $\text{Sec}^2 A = 1 + \tan^2 A,$

whence (7) $\text{Sec } A = \sqrt{1 + \tan^2 A},$

and (8) $\text{Tan}^2 A = \sec^2 A - 1.$

(9) $\text{Tan } A = \sqrt{\sec^2 A - 1}.$

Thirdly, in the right-angled triangle OFH,

$$OH^2 = OF^2 + FH^2,$$

or (10) $\text{Cosec}^2 A = 1 + \cot^2 A;$

and (11) $\text{Cosec } A = \sqrt{1 + \cot^2 A},$

also (12) $\text{Cot}^2 A = \text{cosec}^2 A - 1;$

and (13) $\text{Cot } A = \sqrt{\text{cosec}^2 A - 1}.$

Again,

$$BD = OD - OB,$$

or (14) $\text{Vers } A = 1 - \cos A;$

also $EB = EO + OB,$

or (15) $\text{Suvers } A = 1 + \cos A.$

Therefore (16) $\text{Vers } A \,.\, \text{suvers } A = 1 - \cos^2 A = \sin^2 A.$ (Form. 2.)

In any right-angled triangle where the hypothenuse represents that of ABO, AO being no longer unity,

$$\text{The Sine} = \frac{\text{perpendicular}}{\text{hypothenuse}}.$$

$$\text{Cosine} = \frac{\text{base}}{\text{hypothenuse}}.$$

$$\text{Tangent} = \frac{\text{perpendicular}}{\text{base}}.$$

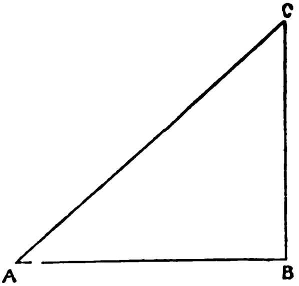
$$\text{Cotangent} = \frac{\text{base}}{\text{perpendicular}}.$$

$$\text{Secant} = \frac{\text{hypothenuse}}{\text{base}}.$$

$$\text{Cosecant} = \frac{\text{hypothenuse}}{\text{perpendicular}}.$$

Therefore in the right-angled triangle ABC,

Fig. 2.



$$\text{Sin A} = \frac{BC}{AC} = \text{cos C}.$$

$$\text{Cos A} = \frac{AB}{AC} = \text{sin C}.$$

$$\text{Tan A} = \frac{BC}{AB} = \text{cot C}.$$

$$\text{Cot A} = \frac{AB}{BC} = \text{tan C}.$$

$$\text{Sec A} = \frac{AC}{AB} = \text{cosec C}.$$

$$\text{Cosec A} = \frac{AC}{CB} = \text{sec C}.$$

And (by Eu. 1, 47),

$$AC^2 = AB^2 + BC^2,$$

dividing both sides by AC^2.

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}.$$

$$1 = \sin^2 A + \cos^2 A;$$

and by dividing by AB^2 and BC^2 in succession, the others already proved may be obtained, which shows them to be true for any triangle.

We know (by Eu. 1, 32), that the three angles of every triangle are equal to two right angles; therefore those of ABC are equal to two right angles; and since $B = 90°$, therefore $A + C = 90°$, or $A = (90° - C)$ and $C = (90° - A)$; or they are complements of each other.

By the definitions already given,

$$\text{Sin A} = \frac{BC}{AC} = \text{cos C} . = \cos(90° - A).$$

$$\therefore \text{Sin A} = \cos(90° - A);$$

or the sine of an angle is the cosine of its complement; and in a similar manner,

$$\text{Tan } A = \cot(90^\circ - A);$$
$$\text{and Sec } A = \text{cosec}(90^\circ - A).$$

From the right-angled triangle ABC,

$$\text{Tan } A = \frac{BC}{AB}.$$

Dividing both numerator and denominator by AC,

$$(1) \quad \text{Tan } A = \frac{\frac{BC}{AC}}{\frac{AB}{AC}} = \frac{\sin A}{\cos A};$$

and in a similar manner,

$$(2) \quad \text{Cot } A = \frac{\cos A}{\sin A}.$$

Again, $\text{Sin } A = \frac{BC}{AC}.$

Dividing by BC,

$$(3) \quad \text{Sin } A = \frac{\frac{BC}{BC}}{\frac{AC}{BC}} = \frac{1}{\text{cosec } A}.$$

And by dividing each time by the numerator, we may prove that

$$(4) \quad \text{Cosec } A = \frac{1}{\sin A}.$$

$$(5) \quad \text{Tan } A = \frac{1}{\cot A}.$$

$$(6) \quad \text{Cot } A = \frac{1}{\tan A}.$$

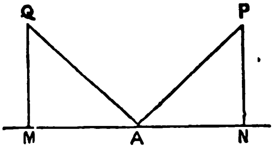
$$(7) \quad \text{Sec } A = \frac{1}{\cos A}.$$

$$(8) \quad \text{Cos } A = \frac{1}{\sec A}.$$

To find the sine and cosine of the supplement of A, or to prove that

$$\text{Sin } A = \sin(180° - A).$$
$$\text{Cos } A = -\cos(180° - A).$$

Fig. 3.



Let the angle PAN = A, then since PAN + PAM = 180°,

$$PAM = (180° - A).$$

Make the angle QAM = A, then QAN = PAM = (180° − A).

Since equal angles have equal cosines and sines,

$$\text{Sin } PAM = \sin QAN.$$
$$\text{Cos } PAM = \cos QAN.$$

Take AQ = AP, and draw QM and PN perpendicular to MN; then QM = PN, and AM = AN; and as AM is opposite in direction to AN, if AM be reckoned positive, AN will be reckoned negative, and

$$\text{Sin } QAN = \frac{QM}{QA} = \frac{PN}{PA} = \sin A;$$
$$\text{or } \sin(180° - A) = \sin A;$$
$$\text{and } \cos QAN = \frac{AM}{AQ} = \frac{-AN}{AP} = -\cos A;$$
$$\text{or } \cos(180° - A) = -\cos A.$$

* * *

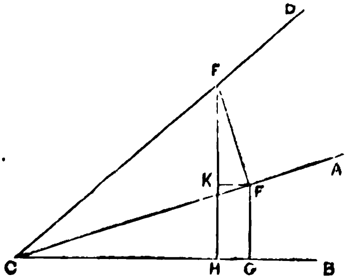
CHAPTER II.

The Sum and Difference of Angles.

To find the sine and cosine of the sum and difference of two angles, or to prove that—

(1) $\text{Sin }(A + B) = \sin A \, . \cos B + \cos A \, . \sin B.$
(2) $\text{Cos }(A + B) = \cos A \, . \cos B - \sin A \, . \sin B.$
(3) $\text{Sin }(A - B) = \sin A \, . \cos B - \cos A \, . \sin B.$
(4) $\text{Cos }(A - B) = \cos A \, . \cos B + \sin A \, . \sin B.$

Fig. 4.



Let the angle ACB = A, and the angle ACD = B, then the angle DCB = (A + B). In CD take any point E, draw EF perpendicular to AC, and FG perpendicular to CB. Draw EH perpendicular to CB, and FK parallel to CB.

Because EFC = 90° (constⁿ.);
Therefore KFC = (90° − EFK).
Because EKF = 90°;
Therefore KEF = (90° − EFK).
But KFC = (90° − EFK);
Therefore KFC = KEF.
But KFC = FCG or A (Eu. 1.29);
Therefore KEF = A.

$$\text{Sin ECH or }(A+B) = \frac{EH}{EC} = \frac{EK+KH}{EC} = \frac{EK+FG}{EC}$$

$$= \frac{EK}{EC} + \frac{FG}{EC} \text{ or } \frac{FG}{EC} + \frac{EK}{EC}.$$

Multiply both numerator and denominator of the first fraction by FC, and the second by EF.

$$\text{Then sin }(A+B) = \frac{FG}{FC}\cdot\frac{FC}{CE} + \frac{EK}{EF}\cdot\frac{EF}{EC}.$$

$$\text{Or sin }(A+B) = \sin A \,.\, \cos B + \cos A \,.\, \sin B.$$

$$\text{Again cos }(A+B) = \frac{CH}{CE} = \frac{CG-GH}{CE} = \frac{CG-KF}{CE}$$

$$= \frac{CG}{CE} - \frac{KF}{CE}.$$

Multiplying as before,

$$\cos(A+B) = \frac{CG}{CF}\cdot\frac{CF}{CE} - \frac{KF}{FE}\cdot\frac{FE}{EC},$$

$$\text{or cos }(A+B) = \cos A \,.\, \cos B - \sin A \,.\, \sin B.$$

Again, since $\sin A = \cos (90^\circ - A)$,
for A write $(A - B)$.

$$\text{Then } \sin (A - B) = \cos (90^\circ - \overline{A - B}) = \cos (\overline{90^\circ - A} + B).$$

Expanding by $\cos (A + B)$,

$$\sin (A - B) = \cos (90^\circ - A) \,.\, \cos B - \sin (90^\circ - A) \sin B;$$

$$\text{or, } \sin (A - B) = \sin A \,.\, \cos B - \cos A \,.\, \sin B.$$

Also since $\cos A = \sin (90^\circ - A)$,
for A write $(A - B)$.

$$\text{Then } \cos (A - B) = \sin (90^\circ - \overline{A - B}) = \sin (\overline{90^\circ - A} + B).$$

Expanding by $\sin (A + B)$,

$$\cos (A - B) = \sin (90^\circ - A) \,.\, \cos B + \cos (90^\circ - A) \sin B;$$

$$\text{or } \cos (A - B) = \cos A \,.\, \cos B + \sin A \,.\, \sin B.$$

N.B.—The first two of these propositions might be proved in a similar way, if we take for granted the truth of the latter two, and these might be proved in a similar way to the first two.

The formulæ dependent upon these are as follow :—

(1) $\text{Sin } 2A = 2 \sin A \,.\, \cos A.$
(2) $\text{Cos } 2A = \cos^2 A - \sin^2 A.$
(3) $\text{Sin } 3A = 3 \sin A - 4 \sin^3 A.$
(4) $\text{Cos } 3A = 4 \cos^3 A - 3 \cos A.$
(5) $\text{Sin } 0^\circ = 0.$
(6) $\text{Cos } 0^\circ = 1.$

Since $\sin (A + B) = \sin A \,.\, \cos B + \cos A \,.\, \sin B$,
for B write A.

$$\text{Then } \sin (A + A) = \sin A \,.\, \cos A + \cos A \,.\, \sin A,$$

$$\text{or (1) } \sin 2A = 2 \sin A \,.\, \cos A.$$

$\text{Cos } (A + B) = \cos A \,.\, \cos B - \sin A \,.\, \sin B$,
for B write A.

$$\text{Then } \cos (A + A) = \cos A \,.\, \cos A - \sin A \,.\, \sin A,$$

$$\begin{aligned} \text{or (2) } \cos 2A &= \cos^2 A - \sin^2 A \\ &= \cos^2 A - (1 - \cos^2 A) \\ &= 2 \cos^2 A - 1 \\ &= 1 - \sin^2 A - \sin^2 A \\ &= 1 - 2 \sin^2 A. \end{aligned}$$

Again, $\sin (A + B) = \sin A . \cos B + \cos A \sin B$,
for B write 2 A.

$$
\begin{aligned}
\text{Then } \sin 3A &= \sin A . \cos 2A + \cos A \sin 2A; \\
\text{and } \sin 3A &= \sin A (1 - 2 \sin^2 A) + \cos A (2 \sin A \cos A) \\
&= \sin A - 2 \sin^3 A + 2 \sin A . \cos^2 A \\
&= \sin A - 2 \sin^3 A + 2 \sin A (1 - \sin^2 A) \\
&= \sin A - 2 \sin^3 A + 2 \sin A - 2 \sin^3 A
\end{aligned}
$$

or, $\sin 3A = 3 \sin A - 4 \sin^3 A$.

Again, $\cos (A + B) = \cos A . \cos B - \sin A . \sin B$,
for B write 2 A.

$$
\begin{aligned}
\text{Then } \cos 3A &= \cos A . \cos 2A - \sin A . \sin 2A, \\
\text{and } \cos 3A &= \cos A . (2 \cos^2 A - 1) - \sin A . (2 \sin A . \cos A) \\
&= 2 \cos^3 A - \cos A - 2 \sin^2 A . \cos A \\
&= 2 \cos^3 A - \cos A - 2 (1 - \cos^2 A) . \cos A \\
&= 2 \cos^3 A - \cos A - 2 \cos A + 2 \cos^3 A;
\end{aligned}
$$

or, $\cos 3A = 4 \cos^3 A - 3 \cos A$.

Again, since $\sin (A - B) = \sin A . \cos B - \cos A . \sin B$,
for B write A.

$$
\begin{aligned}
\text{Then } \sin 0 &= \sin A . \cos A - \cos A \sin A \\
\text{or } \sin 0 &= 0;
\end{aligned}
$$

also since $\cos (A - B = \cos A . \cos B + \sin A \sin B$,
for B write A.

$$
\begin{aligned}
\text{Then } \cos 0 &= \cos A . \cos A + \sin A \sin A \\
&= \cos^2 A + \sin^2 A \\
&= 1.
\end{aligned}
$$

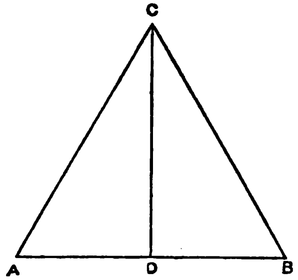
To prove, $\sin 30° = \frac{1}{2} . \cos 30° = \frac{1}{2}\sqrt{3}.$
$\sin 45° = \frac{1}{2}\sqrt{2} = \cos 45°.$
$\sin 60° = \frac{1}{2}\sqrt{3}$ $\cos 60° = \frac{1}{2}.$
$\sin 90° = 1$ $\cos 90° = 0.$
$\sin 180° = 0$ $\cos 180° = -1.$

To find the sin and cos of 30° :—

Let ABC be an equilateral triangle, then each of its angles = 60°.

Bisect the angle ACB by CD, which will bisect the base AB (Eu. 1.4).

Fig. 5.



$$\text{Then } AD = \tfrac{1}{2}\, AB = \tfrac{1}{2}\, AC,$$

$$\text{and } \sin ACD = \sin 30^\circ = \frac{AD}{AC} = \frac{AD}{2\,AD} = \tfrac{1}{2};$$

$$\text{also } \cos 30^\circ = \sqrt{1 - \sin^2 30^\circ} = \sqrt{1 - \tfrac{1}{4}}$$

$$= \sqrt{\tfrac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$= \tfrac{1}{2}\sqrt{3};$$

$$\text{and } \tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

To find the sin, cos, and tan of 45° :—

$$\text{Since } \sin^2 A + \cos^2 A = 1;$$

$$\text{let } A = 45^\circ;$$

$$\text{then } \sin^2 45^\circ + \cos^2 45^\circ = 1,$$

$$\text{or } 2\sin^2 45^\circ = 1,$$

$$\text{and } \sin^2 45^\circ = \tfrac{1}{2},$$

$$\text{or } \sin 45^\circ = \frac{1}{\sqrt{2}} = \tfrac{1}{2}\sqrt{2};$$

$$\text{but } \tan 45^\circ = \frac{\sin 45}{\cos 45^\circ} = 1.$$

To find the sin, cos, and tan of 60° :—

$$\begin{aligned}\text{Since } \sin A &= \cos(90° - A);\\ \text{let } A &= 60;\\ \text{then } \sin 60° &= \cos(90° - 60);\\ \text{or } \sin 60° &= \cos 30° = \tfrac{1}{2}\sqrt{3}.\\ \text{and } \cos 60° &= \sin(90° - 60°)\\ \text{or } \cos 60° &= \sin 30° = \tfrac{1}{2}\end{aligned}$$

$$\text{and } \tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}} = \sqrt{3}.$$

$$\text{and } \sec 60° = \frac{1}{\cos 60°} = \frac{1}{\frac{1}{2}} = 2.$$

To find the tangent of the sum and difference of two angles, in terms of the tangents of the angles, *i.e.*,

$$\text{To prove } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

$$\text{and } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A.\tan B}.$$

$$\text{Tan}(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A.\cos B + \cos A \sin B}{\cos A.\cos B - \sin A \sin B}.$$

Dividing both numerator and denominator by Cos A . cos B

$$= \frac{\dfrac{\sin A.\cos B}{\cos A.\cos B} + \dfrac{\cos A \sin B}{\cos A.\cos B}}{\dfrac{\cos A.\cos B}{\cos A.\cos B} - \dfrac{\sin A \sin B}{\cos A.\cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A.\tan B};$$

$$\text{also, } \tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{\sin A.\cos B - \cos A.\sin B}{\cos A.\cos B + \sin A.\sin B}$$

Dividing as before,

$$= \frac{\dfrac{\sin A.\cos B}{\cos A.\cos B} - \dfrac{\cos A.\sin B}{\cos A.\cos B}}{\dfrac{\cos A.\cos B}{\cos A.\cos B} + \dfrac{\sin A.\sin B}{\cos A.\cos B}}$$

$$= \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

In each formula let $B = 45°$.

Then in the first,

$$\text{Tan}\,(A + 45°) = \frac{\tan A + 1}{1 - \tan A};$$

and in the second,

$$\text{Tan}\,(A - 45°) = \frac{\tan A - 1}{\tan A + 1}.$$

To find the sum and difference of the sines and cosines of two angles.

$$\text{Since } A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B),$$

$$\text{and } B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B),$$

$$\sin A = \sin\{\tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)\}$$

$$\sin B = \sin\{\tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)\}$$

or,

$$\sin A = \sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B) + \cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

and

$$\sin B = \sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B) - \cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

Therefore,

$$(1) \qquad \text{Sin}\,A + \sin B = 2\sin\tfrac{1}{2}(A + B) \,.\, \cos\tfrac{1}{2}(A - B),$$

$$(2) \quad \text{and } \sin A - \sin B = 2\cos\tfrac{1}{2}(A + B) \,.\, \sin\tfrac{1}{2}(A - B).$$

$$\text{Again, Cos}\,A = \cos\{\tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)\}$$

$$\text{and Cos}\,B = \cos\{\tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)\}$$

or,

$$\text{Cos}\,A = \cos\tfrac{1}{2}(A+B) \,.\, \cos\tfrac{1}{2}(A-B) - \sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

and

$$\text{Cos}\,B = \cos\tfrac{1}{2}(A+B) \,.\, \cos\tfrac{1}{2}(A-B) + \sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B).$$

Therefore,

$$(3) \qquad \text{Cos}\,A + \cos B = 2\cos\tfrac{1}{2}(A + B) \,.\, \cos\tfrac{1}{2}(A - B),$$

$$(4) \quad \text{and Cos}\,A - \cos B = -2\sin\tfrac{1}{2}(A - B) \,.\, \sin\tfrac{1}{2}(A - B).$$

If we wish to avoid the negative sign, the latter expression may be written.

$$(4) \ \text{Cos}\,A - \cos B = 2\sin\tfrac{1}{2}(B + A) \,.\, \sin\tfrac{1}{2}(B - A),$$

or, by taking the difference the other way,

$$(4) \ \text{Cos}\,B - \cos A = 2\sin\tfrac{1}{2}(A + B) \,.\, \sin\tfrac{1}{2}(A - B).$$

Dividing (1) by (3),

$$\frac{\text{Sin A} + \sin \text{B}}{\text{Cos A} + \cos \text{B}} = \tan \tfrac{1}{2}(\text{A} + \text{B}).$$

Dividing (2) by (3),

$$\frac{\text{Sin A} - \sin \text{B}}{\text{Cos A} + \cos \text{B}} = \tan \tfrac{1}{2}(\text{A} - \text{B}).$$

CHAPTER III.

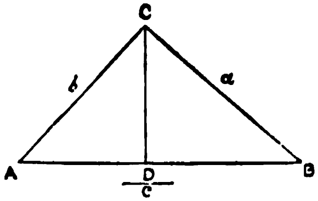
PLANE TRIANGLES.

To find the cosine of an angle of a plane triangle in terms of the sides.

1. When the angle is acute.

Let ABC be a plane triangle, whose angle A is acute. From C draw CD perpendicular to AB. Let the sides of the triangle be a, b, and c respectively.

Fig. 6.



Then (by Eu. 2 . 13)

$$\text{CB}^2 = \text{CA}^2 + \text{AB}^2 - 2\,\text{BA}\,.\,\text{AD}$$

$$\text{or } a^2 = b^2 + c^2 - 2\,c\,.\,\text{AD}\,;$$

$$\text{but AD} = b\,.\cos \text{A}.$$

$$\text{Therefore } a^2 = b^2 + c^2 - 2\,bc \cos \text{A}$$

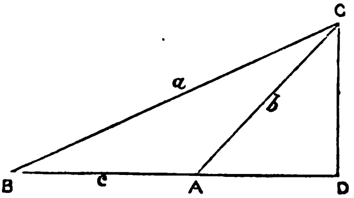
$$\text{and } 2\,bc \cos \text{A} = b^2 + c^2 - a^2$$

$$\text{or} \qquad \cos \text{A} = \frac{b^2 + c^2 - a^2}{2\,bc.}$$

2. When the angle is obtuse.

Produce BA and draw CD perpendicular to BA produced.

Fig. 7.



Then (by Eu. 2. 12),

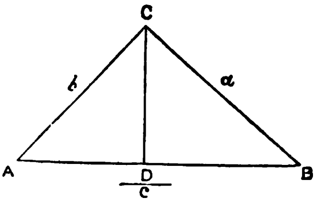
$$BC^2 = BA^2 + AC^2 + 2\,BA\,.\,AD$$
$$\text{or } a^2 = c^2 + b^2 + 2\,c\,.\,AD\,;$$
$$\text{but } AD = AC\,.\cos CAD$$
$$= b\,.\cos CAD$$
$$\text{and } \cos CAD = -\cos CAB = -\cos A.$$
$$\text{Therefore, } a^2 = b^2 + c^2 - 2\,bc\cos A,$$
$$2\,bc\cos A = b^2 + c^2 - a^2,$$
$$\text{and } \cos A = \frac{b^2 + c^2 - a^2}{2\,bc}.$$

Proposition II.

The sides of a plane triangle are to each other as the sines of the opposite angles.

Let ABC be a plane triangle; draw CD perpendicular to AB, and, as before, let the sides be represented by a, b, and c.

Fig. 8.



$$\text{Then } \sin A = \frac{DC}{AC} = \frac{DC}{b}$$
$$\text{and } \sin B = \frac{DC}{BC} = \frac{DC}{a}.$$

Dividing one equation by the other,

$$\frac{\text{Sin A}}{\text{Sin B}} = \frac{\text{DC}}{b} \times \frac{a}{\text{DC}} = \frac{a}{b}.$$

Or, Sin A : sin B : : $a : b$.

From *Proposition* II.

$$\frac{a}{b} = \frac{\sin A}{\sin B}.$$

Add 1 to both sides ;

$$\text{then} \frac{a}{b} + 1 = \frac{\sin A}{\sin B} + 1.$$

$$(1) \quad \text{Or} \frac{a+b}{b} = \frac{\sin A + \sin B}{\sin B}.$$

$$\text{Again,} \frac{a}{b} - 1 = \frac{\sin A}{\sin B} - 1.$$

$$(2) \quad \text{Or,} \frac{a-b}{b} = \frac{\sin A - \sin B}{\sin B}$$

Dividing Eq. (1). by (2),

$$\frac{a+b}{a-b} = \frac{\sin A + \sin B}{\sin A - \sin B}$$

$$= \frac{2 \sin \frac{1}{2}(A+B) . \cos \frac{1}{2}(A-B)}{2 \cos \frac{1}{2}(A+B) . \sin \frac{1}{2}(A-B)}$$

$$\text{or} \quad \frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$$

Proposition IV.

To find an expression for the sine and cosine of half an angle in terms of the sides.

$$\text{Since} \cos A = \frac{b^2 + c^2 - a^2}{2bc},$$

subtract both sides from 1.

$$\text{Then } 1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

$$= \frac{2\,bc - b^2 - c^2 + a^2}{2\,b\,c}.$$

$$= \frac{a^2 - (b - c)^2}{2\,b\,c}.$$

$$= \frac{(a + c - b)\,.\,(a + b - c)}{2\,b\,c}.$$

Let $b + c + a = 2\,S$,
then $a + c - b = 2\,(S - b)$,
and $a + b - c = 2\,(S - c)$.

$$\text{Therefore } 1 - \cos A = \frac{2\,(S - c)\,.\,2\,(S - b)}{2\,b\,c}.$$

But since $\cos 2\,A = \cos^2 A - \sin^2 A$,
and $1 = \cos^2 A + \sin^2 A$;
Therefore $1 + \cos 2\,A = 2\cos^2 A$.
and $1 - \cos 2\,A = 2\sin^2 A$,

$$\text{or} \quad 1 + \cos A = 2\cos^2\frac{A}{2}.$$

$$\text{and} \quad 1 - \cos A = 2\sin^2\frac{A}{2}.$$

Then substituting the value of $1 - \cos A$,

$$2\,.\,\sin^2\frac{A}{2} = \frac{2\,(S - b)\,.\,2\,(S - c)}{2\,b\,c}.$$

$$\text{or } \sin^2\frac{A}{2} = \frac{(S - b)\,.\,(S - c)}{b\,c}.$$

$$\text{and } \sin\frac{A}{2} = \sqrt{\frac{(S - b)\,(S - c)}{b\,c}}.$$

Again, since

$$\text{Cos}\,A = \frac{b^2 + c^2 - a^2}{2\,b\,c},$$

$$1 + \cos A = 1 + \frac{b^2 + c^2 - a^2}{2\,b\,c}$$

$$= \frac{2\,bc + b^2 + c^2 - a^2}{2\,b\,c}$$

$$= \frac{(b + c)^2 - a^2}{2\,b\,c}$$

$$= \frac{(b + c + a)\,(b + c - a)}{2\,b\,c}$$

Substituting as before,

$$2 \cos^2 \frac{A}{2} = \frac{2\,.\,(S - a)\,2\,S}{2\,b\,c},$$

$$\text{or } \cos^2 \frac{A}{2} = \frac{S\,.\,(S - a)}{b\,c},$$

$$\text{and } \cos \frac{A}{2} = \sqrt{\frac{S\,.\,(S - a)}{b\,c}}$$

These two expressions of the sin and cos of half an angle are adapted to logarithms.

$$\text{Again, since } \sin A = 2\,.\,\sin \frac{A}{2}\,.\,\cos \frac{A}{2}.$$

$$\text{and } \sin \frac{A}{2} = \sqrt{\frac{(S - b)\,(S - c)}{b\,c}}.$$

$$\text{Cos } \frac{A}{2} = \sqrt{\frac{S\,(S - a)}{b\,c}}.$$

$$\text{Therefore } \sin A = \frac{2\sqrt{S\,.\,(S - a)\,.\,(S - b)(S - c)}}{b\,c}.$$

$$\text{And } \tan \frac{A}{2} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} = \frac{\sqrt{\frac{(S - b)\,(S - c)}{b\,c}}}{\sqrt{\frac{S\,.\,(S - a)}{b\,c}}}.$$

$$= \sqrt{\frac{(S - b)\,(S - c)}{S\,.\,(S - a)}}.$$

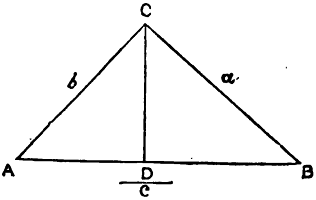
These two are also adapted to logarithms.

Proposition V.

To find an expression for the area of a triangle in terms of the sides.

Let ABC be a triangle, it is required to find an expression for its area.

Fig. 9.



The area of a triangle is equal to half the base multiplied by the perpendicular.

$$\text{Therefore the area of ABC} = \tfrac{1}{2} c \times \text{DC}.$$

$$\text{But DC} = \text{AC} \,.\, \sin \text{A} = b \,.\, \sin \text{A}.$$

$$\text{Therefore area} = \tfrac{1}{2} b c \sin \text{A},$$

which gives the area when the perpendicular is not known.

$$\text{Again } \sin \text{A} = \frac{2\sqrt{\text{S}\,.\,(\text{S}-a)\,(\text{S}-b)\,(\text{S}-c)}}{b\,c}.$$

$$\text{or } b\,c \sin \text{A} = 2\sqrt{\text{S}\,.\,(\text{S}-a)\,(\text{S}-b)\,(\text{S}-c)}.$$

$$\text{and } \tfrac{1}{2} b\,c \sin \text{A} = \sqrt{\text{S}\,.\,(\text{S}-a)\,(\text{S}-b)\,(\text{S}-c)}.$$

$$\text{But } \tfrac{1}{2} b\,c \sin \text{A} = \text{area}.$$

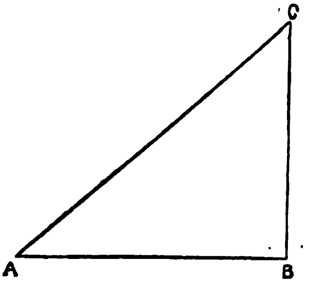
$$\text{Therefore area} = \sqrt{\text{S}\,.\,(\text{S}-a)\,(\text{S}-b)\,(\text{S}-c)}.$$

Right-angled Triangles.

In order to find any part of a right-angled triangle three others must be known, one of which is a side. A triangle consists of seven parts—three sides, three angles, and the area; but the latter will not be considered in the following solutions; it may be found from the above formulæ. The right angle, being always 90°, is considered as a known part, and the following formulæ will furnish us with a solution of any of the cases.

In the right-angled triangle ABC.

Fig. 10.



1. Given AC and angle A, to find the other parts.

$$\left.\begin{array}{r}C = (90^\circ - A)\\ BC = AC \sin A\\ AB = AC \cos A.\end{array}\right\}$$

2. Given AB and angle A.

$$\left.\begin{array}{r}C = (90^\circ - A)\\ BC = AB \,.\, \tan A\\ AC = AB \,.\, \sec A.\end{array}\right\}$$

3. Given BC and angle A.

$$\left.\begin{array}{r}C = (90^\circ - A)\\ AB = BC \cot A\\ AC = CB \,.\, \mathrm{cosec}\, A.\end{array}\right\}$$

4. Given AC and angle C.

$$\left.\begin{array}{r}A = (90^\circ - C)\\ BC = AC \,.\, \cos C\\ AB = AC \sin C.\end{array}\right\}$$

5. Given BC and angle C.

$$\left.\begin{array}{r}A = (90^\circ - C)\\ AB = BC \tan C\\ AC = BC \sec C.\end{array}\right\}$$

6. Given AB and angle C.

$$\left.\begin{array}{r}A = (90^\circ - C)\\ AC = AB \,\mathrm{cosec}\, C\\ BC = AB \cot C.\end{array}\right\}$$

7. Given AC and AB.

$$\left.\begin{array}{c}\frac{AC}{AB} = \mathrm{Sec}\, A \text{ or } \frac{AB}{AC} = \mathrm{Cos}\, A,\\ BC = AC \sin A,\\ \text{or } BC = AB \tan A.\end{array}\right\}$$

8. Given AB and BC.

$$\left.\begin{array}{c}\frac{AB}{BC} = \cot A \text{ or } \frac{BC}{AB} = \tan A,\\ AC = AB \sec A,\\ \text{or } AC = CB \,\mathrm{cosec}\, A.\end{array}\right\}$$

9. Given BC and AC.

$$\left.\begin{array}{c}\frac{BC}{AC} = \sin A \text{ or } \frac{AC}{BC} = \operatorname{cosec} A, \\ AB = BC \cot A, \\ \text{or } AB = AC \cos A.\end{array}\right\}$$

Examples.

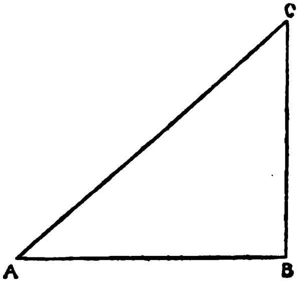
To illustrate what has been said, two examples are worked below, one in each case.

1st. A side and angle being given.

In the right-angled triangle ABC.

$$\left.\begin{array}{l}\text{Given } AC = 840{\cdot}4 \\ \angle A = 38^\circ\ 16'\end{array}\right\}$$

Fig. 11.



Form. 1.

$$C = (90^\circ - A)$$
$$= (90^\circ - 38^\circ\ 16')$$
$$C = 51^\circ\ 44'.$$
$$BC = AC \sin A.$$
$$\operatorname{Log} BC = \log AC + \log \sin A - 10.$$
$$\operatorname{Log} AC = \log 840{\cdot}4 = 2{\cdot}924486.$$
$$\operatorname{Log} \sin A = \log \sin 38^\circ\ 16' = 9{\cdot}791917$$
$$BC\ 520{\cdot}5 = \log \underline{2{\cdot}716403}$$

$$\begin{array}{r}
AB = AC \cos A. \\
\text{Log } AB = \log AC + \log \cos A - 10. \\
\text{Log } AC = \log 840{\cdot}4 = 2{\cdot}924486. \\
\text{Log} \cos A = \log \cos 38^\circ\, 16' = \underline{9{\cdot}894945} \\
AB\ 659{\cdot}8 = \log \underline{\underline{2{\cdot}819431}}
\end{array}$$

2nd. Two sides being given.

$$\text{Given } \left.\begin{array}{r} BC = 520{\cdot}5. \\ AB = 659{\cdot}8. \end{array}\right\}$$

Form. 8.

$$\frac{BC}{AB} = \tan A$$

$$\begin{array}{r}
\log \tan A = \log BC - \log AB + 10. \\
\log BC = \log 520{\cdot}5 = 12{\cdot}716403 \\
\log AB = \log 659{\cdot}8 = \underline{\ 2{\cdot}819431} \\
\angle\ A\ 38^\circ\ 16' \quad \tan \underline{\underline{9{\cdot}896972}}
\end{array}$$

$$\begin{array}{r}
AC = AB\ .\ \sec A. \\
\text{Log } AC = \log AB + \log \sec A - 10. \\
\text{Log } AB = \log 659{\cdot}8 = 2{\cdot}819431 \\
\text{Log} \sec A = \log \sec 38^\circ\ 16' = \underline{10{\cdot}105055} \\
AC\ 840{\cdot}4 \log \underline{\underline{2{\cdot}924486}}
\end{array}$$

Oblique-angled Triangles.

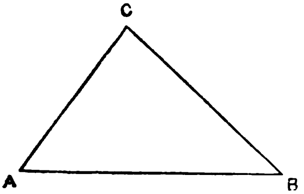
In an oblique-angled triangle three parts are given, in order that the remaining parts may be found. The methods of working the different cases are as follow :—

1. Given two sides and an angle opposite to one of them.

The remainder may be found by Proposition II., which may be written in the form of a rule, thus :

" As any side of a plane triangle is to the sine of its opposite angle, so is any other side of the same triangle to the sine of its opposite angle."

Fig. 12.



Or, in the plane triangle ABC, given AB, AC, and the angle B, to find the other parts.

Sin C : AB :: Sin B : AC

$$\text{whence } \sin C = \frac{AB . \sin B}{AC}$$

which gives C.

Again $A = 180° - (B + C)$

and $\sin C : AB :: \sin A : BC$

$$\text{whence } BC = \frac{\sin A . AB}{\sin C}$$

which gives BC

2. Given two sides and the included angle, to find the rest.

Proposition III. may here be written as a rule.

" As the sum of the given sides is to their difference, so is the tangent of half the sum of the opposite angles to the tangent of half their difference. And the half difference added to half the sum gives the angle opposite the greater side, and, subtracted, gives the angle opposite the less."

Given AC, CB, and angle C, to find the other parts.

Then by Proposition III.

$$(a + b) : (a - b) :: \tan \tfrac{1}{2} (A + B) : \tan \tfrac{1}{2} (A - B),$$

whence $\tan \frac{1}{2} (A - B)$ may be found, and

$$\text{tangent of } \tfrac{1}{2} (A + B) = \tan \frac{180° - C}{2}$$

$$\text{Then } \tfrac{1}{2} (A + B) + \tfrac{1}{2} (A - B) = A.$$
$$\text{and } \tfrac{1}{2} (A + B) - \tfrac{1}{2} (A - B) = B.$$

When A and B are found, AB may be obtained as in the last case.

3. Given three sides, a, b, c.

To find angle A.

$$\text{Cos} \frac{A}{2} = \sqrt{\frac{S (S - a)}{bc}}$$

To find angle B.

$$\text{Cos} \frac{B}{2} = \sqrt{\frac{S . (S - b)}{ac}}$$

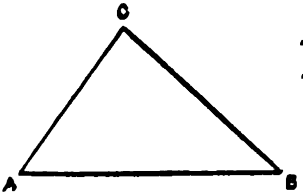
To find angle C.

$$\operatorname{Cos}\frac{C}{2}=\sqrt{\frac{S.(S-c)}{ab}}.$$

S being the sum of the three sides.

Examples.

Fig. 18.



In the oblique-angled triangle ABC,

Given BC = 460
AC = 654
∠A = 35° 12′

To find angle B.

$$BC : \sin A :: AC : \sin B.$$

$$\text{Whence } \sin B = \frac{AC\,.\,\sin A}{BC}.$$

And log sin B = log AC + log sin A − log BC.

Log AC = log 654 = 2·815578
log sin A = sin 35° 12′ = 9·760748
12·576326
log BC = log 460 = 2·662758
B 55° 2′ sin 9·913568

A = 35° 12′
B = 55 2
90 14
180 0
C = 89 46

To find AB.

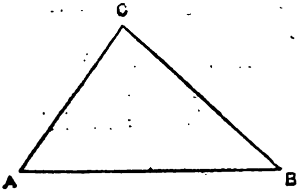
$$\operatorname{Sin} B : AC :: \sin C : AB$$

$$\text{Whence } AB = \frac{AC\,.\,\sin C}{\sin B}$$

and log AB = log AC + log sin C − log sin B

$$\begin{array}{rr}
\log AC = \log 654 = & 2{\cdot}815575 \\
\log \sin C = \sin 89^\circ\, 46' = & \underline{9{\cdot}999996} \\
 & 12{\cdot}815571 \\
\sin B = \sin 55^\circ\, 2' = & \underline{9{\cdot}913568} \\
AB\ 798 = & \underline{\underline{2{\cdot}902003}}
\end{array}$$

Fig. 14.



In the oblique-angled triangle ABC.

Given AC = 534
CB = 848
angle C = 31° 17′
To find $\frac{1}{2}(A + B)$.

$$\begin{array}{r}
C = 31^\circ\ 17' \\
\underline{180\quad 0\ } \\
2)\underline{148\quad 43} \\
\frac{1}{2}(A+B)\ \underline{\underline{74\quad 21}}
\end{array}$$

$$(a + b) : (a - b) :: \tan \tfrac{1}{2}(A + B) : \tan \tfrac{1}{2}(A - B),$$

or $(848 + 534) : (848 - 534) :: \tan 74^\circ\, 21' : \tan \frac{1}{2}(A - B)$

$$\therefore \tan \tfrac{1}{2}(A-B) = \frac{314 \times \tan 74^\circ\, 21'}{1382}.$$

$$\begin{array}{rr}
\log 314 = & 2{\cdot}496930 \\
\tan 74^\circ 21' = & \underline{10{\cdot}552616} \\
 & 13{\cdot}049546 \\
\log 1382 = & \underline{3{\cdot}140508} \\
\frac{1}{2}(A-B)\ 39^\circ\ 2' \quad \tan & \underline{\underline{9{\cdot}909038}}
\end{array}$$

$$\begin{array}{r}
\frac{1}{2}(A - B) = 39^\circ\ 2' \\
\frac{1}{2}(A + B) = \underline{74\quad 21} \\
A\ \underline{113\quad 23} \\
B\ \underline{\underline{35\quad 19}}
\end{array}$$

The remaining part, AB, may be found as in the last example.

CHAPTER IV.

Spherical Trigonometry.

The definition of a sphere, and of great and less circles, having been given in Part I., Chapter 19, it is sufficient here to state that :—

1. A sphere may be conceived to be generated by the revolution of a semicircle about its diameter.

2. The pole of a circle is a point on the surface of the sphere, 90° distant from every point in the circumference of the circle.

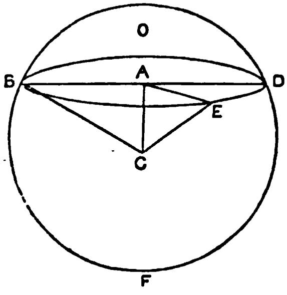
3. A spherical triangle is one formed on the surface of the sphere, by the intersection of three planes which meet in the centre of the sphere.

Proposition I.

Every section of a sphere by a plane is a circle.

Let the sphere BFD be cut so as to expose the plane section BOED ; it is required to show that this section is a circle. Join BD. Find C, the centre of the sphere, and in the circumference of the plane surface take a point E, join CB and CE, draw CA perpendicular to BD, and join AE.

Fig. 15.



If the plane pass through the centre, then the section is a figure, every point in whose boundary is equidistant from the centre of the sphere ; it is therefore a circle.

But if the plane does not pass through the centre, then (the above construction being made),—

In the two triangles CAB and CAE ;

CB = CE (Def^n.)

and the angle CAB = the angle CAE (each 90°),

and CA is common ;

therefore BA = AE.

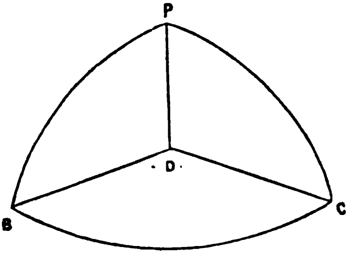
In a similar manner it might be shown that AE = AD, therefore BA, AE, and AD are all equal; and therefore the figure is a circle.

Proposition II.

The angle at the pole of a great circle is measured by the arc which subtends it.

Let P be the pole of a circle, an arc of which is BC; it is required to show that the angle P is measured by BC. Find D, the centre of the sphere. Join DB, DC, and DP.

Fig. 16.

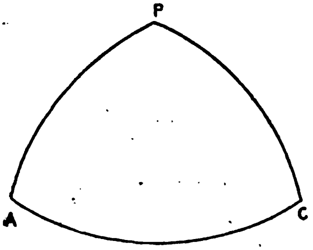


The solid angle at D is measured by the inclination of the planes PDB and PDC, and the angle BPC is measured by it. But the arc BC measures the angle BDC; therefore it measures the angle BPC.

Proposition III.

The distance of a great circle from its pole is a quadrant.

Fig. 17.



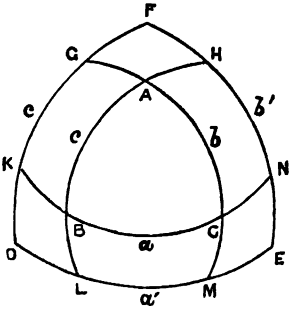
Let P be the pole of a great circle whose arc is AC, and APC a great circle passing through P.

Then, as P is equally distant from every point in AC, PA and PC are quadrants (for all great circles bisect each other).

Proposition IV.

The polar triangle.

Fig. 18.



Let ABC be a spherical triangle. Take B as a centre, and, with a radius of 90°, describe the arc FE; in a similar manner describe the arcs DE and DF; then DEF is the polar triangle.

Produce BC both ways to K and N, AB to L and H, AC to G and M.

1. The angular points of the polar triangle are the poles of the sides of the primitive triangle.

To prove that F is the pole of BC.

Since B is the pole of EF,

$$BH = 90^\circ;$$

and since C is the pole of FD,

$$CG = 90^\circ.$$

Therefore BH and CG are each equal to 90°,

and F is 90° distant from BC,

or, it is the pole of BC.

In a similar manner it might be shown that E is the pole of AB, and D the pole of AC.

2. The sides and angles of the polar triangle are the supplements of the angles and sides of the primitive triangle.

To prove $A = (180^\circ - DE)$.

$$DE = DM + ME;$$

$$\text{but } ME = EL - LM.$$

$$\text{Therefore } DE = DM + EL - LM;$$

$$\text{But } DM = 90^\circ \text{ and } LE = 90^\circ.$$

$$\text{Therefore } DE = 180^\circ - LM;$$

and LM measures the angle A.

$$\text{Therefore } DE = (180^\circ - A).$$

In a similar way it might be shown that $FE = (180^\circ - B)$, and $FD = (180^\circ - C)$.

$$\text{Again, } KN = KC + CN;$$
$$\text{but } CN = NB - BC.$$
$$\text{Therefore } KN = KC + NB - BC;$$
$$\text{but } KC = 90^\circ, \text{ and } NB = 90^\circ.$$
$$\text{Therefore } KN = (180^\circ - BC);$$
$$\text{but BC measures the angle F.}$$
$$\text{Therefore } KN = (180^\circ - F).$$
$$\text{In a similar manner, } GM = (180^\circ - D)$$
$$\text{and } LH = (180^\circ - E).$$

Proposition V.

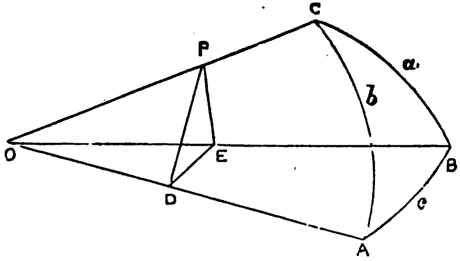
To find the cosine of an angle of a spherical triangle in terms of the cosines of the sides.

Let ABC be a spherical triangle: take O, the centre of the sphere, join OA, OB, and OC.

In OC take any point, P, draw PD perpendicular to OA, in the plane OAC.

From D draw DE perpendicular to OA in the plane OAB, and join PE.

Fig. 19.



Then the solid angle at O is composed of the three angles COA, COB, and AOB, which are measured by the arcs *b*, *a*, and *c* respectively.

$$\text{Then } PE^2 = PO^2 + OE^2 - 2\,PO\,.\,OE \cos POE,$$
$$\text{and } PE^2 = PD^2 + DE^2 - 2\,PD\,.\,DE \cos PDE;$$

but $\cos POE = \cos a$

and $\cos PDE = \cos A$, for the angle A is measured by the inclination of the planes COA and AOB.

$$\text{Therefore } PE^2 = PO^2 + OE^2 - 2\,PO\,.\,OE \cos a$$
$$\text{and } PE^2 = PD^2 + DE^2 - 2\,PD\,.\,DE \cos A.$$

and subtracting—

$$O = PO^2 - PD^2 + OE^2 - ED^2 - 2\,PO\,.\,OE\,.\cos a$$
$$+ 2\,PD\,.\,DE \cos A;$$

$$\text{but } PO^2 - PD^2 = OD^2 \text{ (PDO being } 90^\circ\text{)},$$
$$\text{and } OE^2 - ED^2 = OD^2 \text{ (ODE being } 90^\circ\text{)}.$$
$$\text{Then } 0 = OD^2 + OD^2 - 2\,PO\,.\,OE \cos a + 2\,PD\,.\,DE \cos A$$
$$= 2\,OD\,.\,OD - 2\,PO\,.\,OE \cos a + 2\,PD\,.\,DE \cos A$$
$$\text{and } 2\,PO\,.\,OE \cos a = 2\,OD\,.\,OD + 2\,PD\,.\,DE \cos A.$$

Dividing by 2 OP . OE,

$$\cos a = \frac{OD\,.\,OD}{OP\,.\,OE} + \frac{PD\,.\,DE}{PO\,.\,OE}\,.\cos A$$
$$= \cos b\,.\cos c + \sin b\,.\sin c \cos A\,;$$
$$\text{and } \cos a - \cos b\,.\cos c = \sin b\,.\sin c \cos A.$$
$$\text{Therefore } \frac{\cos a - \cos b\,.\cos c}{\sin b\,.\sin c} = \cos A.$$

N.B.—Compare this with the value of cos A in a plane triangle.

Proposition VI.

To find the sine and cosine of ½ A in terms of the sines and cosines of the sides.

$$\text{Since } \cos A = \frac{\cos a - \cos b\,.\cos c}{\sin b\,.\sin c}$$
$$1 + \cos A = 1 + \frac{\cos a - \cos b\,.\cos c}{\sin b\,.\sin c}$$
$$= \frac{\sin b\,.\sin c + \cos a - \cos b\,.\cos c}{\sin b\,.\sin c}$$
$$= \frac{\cos a - (\cos b\,.\cos c - \sin b\,.\sin c)}{\sin b\,.\sin c}$$
$$= \frac{\cos a - \cos (b + c)}{\sin b\,.\sin c}$$
$$= \frac{2 \sin \frac{1}{2}(b + c + a) \sin \frac{1}{2}(b + c - a)}{\sin b\,.\sin c}.$$

$$\text{Let } b + c + a = 2\,S$$
$$\text{then } \tfrac{1}{2}(b + c + a) = S$$
$$\text{and } \tfrac{1}{2}(b + c - a) = (S - a)$$
$$\text{and } 1 + \cos A = 2 \cos^2 \frac{A}{2}.$$

$$\text{Therefore } 2\cos^2\frac{A}{2} = \frac{2\sin S \,.\, \sin(S-a)}{\sin b \,.\, \sin c}$$

$$\text{and } \cos\frac{A}{2} = \sqrt{\frac{\sin S \,.\, \sin(S-a)}{\sin b \,.\, \sin c}}.$$

$$\text{Again, } 1 - \cos A = 1 - \frac{\cos a - \cos b \,.\, \sin c}{\sin b \,.\, \sin c}$$

$$= \frac{\sin b \,.\, \sin c - \cos a + \cos b \,.\, \cos c}{\sin b \sin c}$$

$$= \frac{\cos(b-c) - \cos a}{\sin b \sin c}$$

$$= \frac{2\sin\frac{1}{2}(a+c-b)\sin\frac{1}{2}(a+b-c)}{\sin b \,.\, \sin c}.$$

Then, as before, if $(b+c+a) = 2\,S$

$$2\sin^2\frac{A}{2} = \frac{2\sin(S-b)\sin(S-c)}{\sin b \,.\, \sin c}$$

$$\text{and } \sin\frac{A}{2} = \sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin b \,.\, \sin c}}.$$

$$\text{Also, since } \sin A = 2\sin\frac{A}{2} \,.\, \cos\frac{A}{2}$$

$$= 2\sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin b \sin c}}\sqrt{\frac{\sin S \,.\, \sin(S-a)}{\sin b \sin c}};$$

$$\text{therefore } \sin A = \frac{2\sqrt{\sin S \,.\, \sin(S-a)\sin(S-b)\sin(S-c)}}{\sin b \,.\, \sin c}$$

$$\text{and } \tan\frac{A}{2} = \sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin S \,.\, \sin(S-a)}}$$

Proposition VII.

Napier's Analogies.

$$\text{Since } \cos A = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}$$

and $\cos c = \cos a \,.\, \cos b + \sin a \sin b \cos C.$

substituting,

$$\text{Cos } A = \frac{\cos a - \cos b\,(\cos a \cos b + \sin a \sin b \cos C)}{\sin b \,.\, \sin c}$$

$$=\frac{\cos a-\cos a\cos^2 b-\sin a\sin b\,.\cos b\,.\cos C}{\sin b\,.\sin c}$$

$$=\frac{\cos a\,(1-\cos^2 b)-\sin a\sin b\,.\cos b\,.\cos C}{\sin \mathrm{b}\,.\sin c}$$

$$=\frac{\cos a\sin^2 b-\sin a\sin b\,.\cos b\,.\cos C}{\sin b\sin c}$$

$$(1)\quad \text{Cos } A=\frac{\cos a\sin b-\sin a\cos b\,.\cos C}{\sin c}$$

In a similar manner,

$$(2)\quad \text{Cos } B=\frac{\cos b\,.\sin a-\sin b\,.\cos a\cos C}{\sin c.}$$

Adding (1) and (2)

$$(3)\quad \text{Cos } A+\text{Cos } B=\frac{\sin(a+b)-\sin(a+b)\cos C}{\sin c}$$

$$=\frac{\sin(a+b)\,(1-\cos C)}{\sin c}$$

$$\text{But } \sin A=\frac{\sin a}{\sin c}.\sin C$$

$$\text{and } \sin B=\frac{\sin b}{\sin c}.\sin C.$$

$$(4)\quad \text{Therefore } \sin A+\sin B=\frac{\sin a+\sin b}{\sin c}.\sin C.$$

Dividing (4) by (3),

$$\frac{\text{Sin } A+\sin B}{\text{Cos } A+\cos B}=\frac{\sin a+\sin b}{\sin(a+b)}\cdot\frac{\sin C}{2\sin^2\frac{C}{2}}$$

$$\text{But } \frac{\sin A+\sin B}{\cos A+\cos B}=\tan\tfrac{1}{2}(A+B).$$

$$\text{Therefore } \tan\tfrac{1}{2}(A+B)=\frac{2\sin\frac{1}{2}(a+b)\cos\frac{1}{2}(a-b)}{2\sin\frac{1}{2}(a+b)\cos\frac{1}{2}(a+b)}\cdot\frac{\sin C}{2\sin^2\frac{C}{2}}$$

$$=\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cdot\frac{\sin C}{2\sin^2\frac{C}{2}}$$

$$\text{But } \frac{\sin C}{2\sin^2\frac{C}{2}} = \frac{2\sin\frac{C}{2}\cdot\cos\frac{C}{2}}{2\sin^2\frac{C}{2}} = \cot\frac{C}{2}.$$

$$(1)\qquad \operatorname{Tan}\tfrac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cdot\cot\frac{C}{2}.$$

$$\text{Again } \sin A - \sin B = \frac{\sin a - \sin b}{\sin c}\cdot\sin C$$

$$\text{and } \frac{\sin A - \sin B}{\cos A + \cos B} = \frac{\sin a - \sin b}{\sin(a+b)}\cdot\frac{\sin C}{2\sin^2\frac{C}{2}};$$

$$\text{but } \frac{\sin A - \sin B}{\cos A + \cos B} = \tan\tfrac{1}{2}(A-B).$$

$$\text{Therefore } \tan\tfrac{1}{2}(A-B) = \frac{2\cos\frac{1}{2}(a+b)\sin\frac{1}{2}(a-b)}{2\sin\frac{1}{2}(a+b)\cos\frac{1}{2}(a+b)}\cdot\frac{\sin C}{2\sin^2\frac{C}{2}}.$$

$$(2)\text{ Then }\ \tan\tfrac{1}{2}(A-B) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cdot\cot\frac{C}{2}.$$

These are Napier's first and second analogies.

Napier's Third and Fourth Analogies.

$$\text{Since } \tan\tfrac{1}{2}(A+B) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\frac{C}{2}.$$

If this expression be true for any triangle, it is true for the Polar or Supplemental Triangle, whose angles may be represented by A', B', and C', and corresponding sides a', b', and c'.

$$\text{Then } \tan\tfrac{1}{2}(A'+B') = \frac{\cos\frac{1}{2}(a'-b')}{\cos\frac{1}{2}(a'+b')}\cdot\cot\frac{C'}{2}$$

$$\begin{aligned}\text{But } \tfrac{1}{2}(A'+B') &= \tfrac{1}{2}\{\pi - a + \pi - b\} = \tfrac{1}{2}\{2\pi - \overline{a+b}\}\\ &= \pi - \tfrac{1}{2}(a+b),\end{aligned}$$

$$\begin{aligned}\text{and } \tfrac{1}{2}(a'-b') &= \tfrac{1}{2}(\pi - A - \overline{\pi - B})\\ &= \tfrac{1}{2}(B-A),\end{aligned}$$

$$\begin{aligned}\text{and } \tfrac{1}{2}(a'+b') &= \tfrac{1}{2}(\pi - A + \pi - B) = \tfrac{1}{2}(2\pi - (A+B))\\ &= \pi - \tfrac{1}{2}(A+B).\end{aligned}$$

Then substituting

$$\text{Tan}\,(\pi - \tfrac{1}{2}\,\overline{a+b}) = \frac{\text{Cos}\,\frac{1}{2}\,(\text{B}-\text{A})}{\text{Cos}\,(\pi - \frac{1}{2}\,\overline{\text{A}+\text{B}})}\,.\,\cot\frac{\text{C}'}{2}.$$

$$\text{But}\ \cot\frac{\text{C}'}{2} = \tan\frac{c}{2},$$

$$\text{and}\ \tan\,(\pi - \tfrac{1}{2}\,\overline{a+b}) = -\tan\tfrac{1}{2}\,(a+b),$$

$$\text{and}\ \cos\tfrac{1}{2}\,(\text{B}-\text{A}) = \cos\tfrac{1}{2}\,(\text{A}-\text{B}),$$

$$\text{and}\ \cos\,(\pi - \tfrac{1}{2}\,(\text{A}+\text{B})) = -\cos\tfrac{1}{2}\,(\text{A}+\text{B}).$$

Therefore

$$-\tan\tfrac{1}{2}\,(a+b) = \frac{\cos\frac{1}{2}\,(\text{A}-\text{B})}{-\cos\frac{1}{2}\,(\text{A}+\text{B})}\,.\,\tan\frac{c}{2}.$$

Or, dividing both sides by − 1,

(3) $$\text{Tan}\,\tfrac{1}{2}\,(a+b) = \frac{\cos\frac{1}{2}\,(\text{A}-\text{B})}{\cos\frac{1}{2}\,(\text{A}+\text{B})}\,.\,\tan\frac{c}{2}.$$

Again, since

$$\text{Tan}\,\tfrac{1}{2}\,(\text{A}'-\text{B}') = \frac{\sin\frac{1}{2}\,(a'-b')}{\sin\frac{1}{2}\,(a'+b')}\,.\,\cot\frac{\text{C}'}{2}$$

$$\begin{aligned}\tfrac{1}{2}\,(\text{A}'-\text{B}') = \tfrac{1}{2}\,(\pi - a - \overline{\pi - b}) &= \tfrac{1}{2}\,(b-a)\\ &= -\tfrac{1}{2}\,(a-b),\end{aligned}$$

$$\text{also}\ \tfrac{1}{2}\,(a'-b') = -\tfrac{1}{2}\,(\text{A}-\text{B}),$$

$$\text{and}\ \tfrac{1}{2}\,(a'+b') = \tfrac{1}{2}\,(\text{A}+\text{B}),$$

$$\text{and}\ \cot\frac{\text{C}'}{2} = \tan\frac{c}{2},$$

Then

$$-\tan\tfrac{1}{2}\,(a-b) = \frac{-\sin\frac{1}{2}\,(\text{A}-\text{B})}{\sin\frac{1}{2}\,(\text{A}+\text{B})}\,.\,\tan\frac{c}{2},$$

and multiplying both sides by − 1,

(4.) $$\text{Tan}\,\tfrac{1}{2}\,(a-b) = \frac{\sin\frac{1}{2}\,(\text{A}-\text{B})}{\sin\frac{1}{2}\,(\text{A}+\text{B})}\,.\,\tan\frac{c}{2}.$$

These are Napier's third and fourth analogies.

Proposition VIII.

To find the cosine of a side of a spherical triangle in terms of the angles.

$$\text{Since } \cos A = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}.$$

By referring to the polar triangle,

$$\cos A' = \frac{\cos a' - \cos b' \cos c'}{\sin b' \sin c'};$$

$$\text{but } A' = (\pi - a)$$
$$a' = (\pi - A)$$
$$b' = (\pi - B)$$
$$c' = (\pi - C)$$

$$\text{Therefore, } \cos(\pi - a) = \frac{\cos(\pi - A) - \cos(\pi - B)\cos(\pi - C)}{\sin(\pi - B)\sin(\pi - C)}$$

$$\text{or, } -\cos a = \frac{-\cos A - (-\cos B)\,.\,(-\cos C)}{\sin B \,.\, \sin C}$$

$$= \frac{-\cos A - \cos B \,.\, \cos C}{\sin B \sin C}.$$

Multiplying by -1,

$$\text{then, } \cos a = \frac{\cos A + \cos B \,.\, \cos C}{\sin B \sin C}.$$

Proposition IX.

To find a formula which shall connect four successive parts of a spherical triangle, or to show that—

$$\text{Cot } a \sin b = \cot A \,.\, \sin C + \cos C \cos b.$$

$$\text{Since } \cos a = \frac{\cos A + \cos B \,.\, \cos C}{\sin B \,.\, \sin C},$$

$$\cos a \sin B \sin C = \cos A + \cos B \,.\, \cos C.$$

$$\text{But } \sin B = \sin A \,.\, \frac{\sin b}{\sin a}$$

$$\text{and } \cos B = \sin A \sin C \cos b - \cos A \,.\, \cos C.$$

Substituting these values :—

$$\begin{aligned}\text{Cot } a \sin b \sin A \sin C &\\ &= \cos A + \sin A \sin C \cos C \,.\, \cos b - \cos A \cos^2 C\\ &= \cos A \sin^2 C + \sin A \sin C \cos C \cos b.\end{aligned}$$

And dividing by sin A sin C,

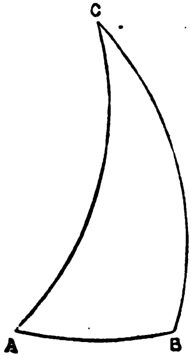
$$\cot a \sin b = \cot A \sin C + \cos b \,.\, \cos C\,;$$

which may be written also,

$$\cot A \sin C = \cot a \sin b - \cos C \cos b.$$

Napier's Rules for the Solution of Right-angled Spherical Triangles.

Fig. 20.



Let ABC be a spherical triangle; B the right angle; a, b, and c the sides, of which b is the hypothenuse. Then, if we take two sides, the complements of the angles, and the complement of the hypothenuse, or (90° − A), (90° — C) and (90° − b), and call any one of them a middle part, and the two nearest (one on each side) adjacent parts, and the other two which are more distant, extreme parts, we shall have two general rules, viz. :—

Sine of the middle part = prod. of tangents of adjacents ; and,

Sine of the middle part = prod. of cosines of extremes.

Thus, if we take (90° − b) as the middle part, (90° — A) and (90° — C) are the adjacents, and a and c are the extremes. Then following the rule,

$$\begin{aligned}\text{Sin } (90^\circ - b) &= \tan (90^\circ - A) \,.\, \tan (90^\circ - C),\\ \text{or, } \cos b &= \cot A \,.\, \cot C\,.\end{aligned}$$

This, together with the others below mentioned, we shall now investigate.

$$\left.\begin{array}{ll}(1.) & \text{Cos } b = \cos a \cos c\\ (2.) & \text{Cos } b = \cot A \,.\, \cot C\end{array}\right\}$$

$$\left.\begin{array}{ll}(3.) & \text{Cos } A = \cos a \sin C\\ (4.) & \text{Cos } A = \cot b \,.\, \tan c\end{array}\right\}$$

$$\left.\begin{array}{ll}(5.) & \text{Sin } c = \sin b \,.\, \sin C\\ (6.) & \text{Sin } c = \cot A \,.\, \tan a\end{array}\right\}$$

To prove (1) and (2) :—

$$\text{Since } \cos B = \frac{\cos b - \cos a \cos c}{\sin a \sin c}$$

$$\text{and } B = 90°, \text{ therefore } \cos B = 0.$$

$$\text{Therefore, } 0 = \frac{\cos b - \cos a \cos c}{\sin a \sin c}$$

$$\text{or, } 0 = \cos b - \cos a \cos c$$

$$(1) \quad \text{and } \cos b = \cos a \cos c\,;$$

Again,

$$\cos b = \frac{\cos B + \cos A \,.\, \cos C}{\sin A \,.\, \sin C}$$

$$B = 90° \text{ and } \cos B = 0\,.$$

$$(2) \quad \text{Therefore } \cos b = \cot A \,.\, \cot C\,.$$

To prove (3) and (4) :—

$$\text{Since } \cos a = \frac{\cos A + \cos B \,.\, \cos C}{.\, \sin B \,.\, \sin C}$$

$$\cos B = 0, \ \sin B = 1\,;$$

$$\text{therefore, } \cos a = \frac{\cos A}{\sin C}$$

$$(3) \quad \text{or, } \cos A = \cos a \sin C\,.$$

Again,

$$\text{Cos } A = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c}$$

$$\text{but } \cos a = \frac{\cos b}{\cos c}\,;$$

$$\text{therefore } \cos A = \frac{\cos b - \cos b \,.\, \cos^2 c}{\sin b \sin c \cos c}$$

$$= \frac{\cos b\,(1 - \cos^2 c)}{\sin b \sin c \cos c}$$

$$= \frac{\cos b \,.\, \sin^2 c}{\sin b \sin c \cos c}$$

$$= \frac{\cos b \,.\, \sin c}{\sin b \cos c}\,;$$

$$(4) \quad \text{or } \cos A = \cot b \,.\, \tan c\,.$$

To prove (5) and (6) :—

$$\frac{\sin b}{\sin c} = \frac{\sin B}{\sin C}.$$

$$\sin B = 1.$$

$$\text{Therefore, } \frac{\sin b}{\sin c} = \frac{1}{\sin C};$$

$$(5) \quad \text{or, } \sin c = \sin b \,.\, \sin C.$$

Again,

$$\cot A \,.\, \sin B = \cot a \sin c - \cos B \cos c \text{ (Prop. IX.)},$$

$$\sin B = 1 \text{ and } \cos B = 0.$$

$$\text{Therefore, } \cot A = \cot a \sin c,$$

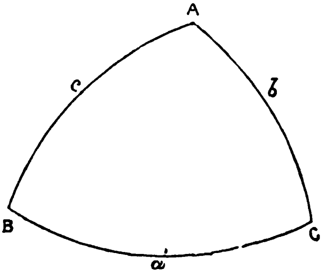
$$(6) \quad \text{or, } \sin c = \cot A \tan a.$$

Oblique Angled Spherical Triangles.

1. Given two sides of a spherical triangle, and the included angle, to find the third side.

Let a, b, and angle C be given.

Fig. 21.



$$\text{Since } \cos c = \cos a \cos b + \sin a \sin b \,.\, \cos C,$$

$$\text{and } \cos C = (2 \cos^2 \frac{C}{2} - 1);$$

therefore,

$$\cos c = \cos a \cos b + \sin a \sin b \,.\, (2 \cos^2 \frac{C}{2} - 1)$$

$$= \cos a \cos b - \sin a \sin b + 2 \sin a \sin b \cos^2 \frac{C}{2}$$

$$= \cos (a + b) + 2 \sin a \sin b \,.\, \cos^2 \frac{C}{2};$$

and

$$1 - \cos c = 1 - \cos (a + b) - 2 \sin a \sin b \,.\, \cos^2 \frac{C}{2};$$

or,

$$2 \sin^2 \frac{c}{2} = 2 \sin^2 \frac{a + b}{2} - 2 \sin a \sin b \,.\, \cos^2 \frac{C}{2}.$$

$$\text{Let } \sin \theta = \sin a \sin b \cos^2 \frac{C}{2}.$$

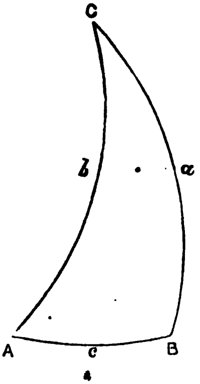
$$\text{Then } \sin^2 \frac{c}{2} = \sin^2 \frac{a + b}{2} - \sin^2 \theta.$$

$$\sin^2 \frac{C}{2} = \sin \left\{\frac{a + b}{2} + \theta\right\} \sin \left\{\frac{a + b}{2} - \theta\right\}.$$

2. Given two sides and an opposite angle, or two angles and an opposite side.

This case depends on the following proposition.

Fig 22.



In any spherical triangle, the sines of the sides are proportional to the sines of the opposite angles.

In the right-angled triangle ABC,

$$\sin AB = \sin AC \,.\, \sin C,$$

$$\text{and } \sin BC = \sin AC \,.\, \sin A.$$

$$\text{Therefore, } \frac{\sin AB}{\sin BC} = \frac{\sin C}{\sin A},$$

or, $\sin c : \sin a :: \sin C : \sin A$.

Examples.

In the right-angled spherical triangle (*see* last fig.) ABC, given AB = 46° 18′ 23″ and angle A = 34° 27′ 39″, to find the other parts.

Here the angle A is acute; therefore BC is also acute, and as AB is acute, AB and BC are of the same affection (*i.e.*, they are both acute), therefore AC and angle C are also less than 90°.

To compute BC.

$$\sin AB = \cot A \,.\, \tan BC \;(6),$$
$$\text{whence } \tan BC = \sin AB \tan A.$$
$$\sin AB = 9 \cdot 859165$$
$$\tan A = 9 \cdot 836497$$
$$BC\ 29^\circ\ 33'\ 29''\ \tan\ 9 \cdot 695662$$

To compute AC.

$$\cos A = \cot AC \tan AB,$$
$$\text{whence } \cot AC = \cos A \cot AB$$
$$\cos A = 9 \cdot 916198$$
$$\cot AB = 9 \cdot 980189$$
$$AC\ 51^\circ\ 46'\ 15''\ \cot\ 9 \cdot 896387$$

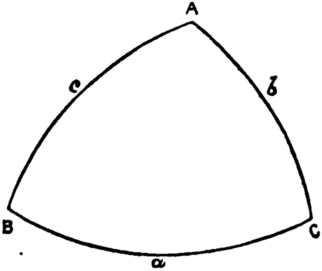
To compute C.

$$\cos C = \cos AB \sin A$$
$$\cos AB = 9 \cdot 839354$$
$$\sin A = 9 \cdot 752695$$
$$C\ 66^\circ\ 59'\ 26''\ \cos\ 9 \cdot 592049$$

N.B.—A formula should be taken which contains the two given parts; it is not necessary to use one which will cause the result to depend upon the computed parts.

In the oblique-angled triangle ABC, given AB = 40° 37′, BC = 91° 8′, and angle B = 58° 31′; to find the other parts.

Fig 23.



$$\sin^2 \theta = \sin AB \,.\, \sin BC \,.\, \cos^2 \tfrac{1}{2} B.$$

$$\sin^2 \frac{AC}{2} = \sin \left\{\frac{AB + BC}{2} + \theta\right\} \sin \left\{\frac{AB + BC}{2} - \theta\right\}$$

$$\begin{array}{rl} & \quad {}^\circ \quad\ \ ' \\ B = & 2\overline{)58\ \ 31} \\ \tfrac{1}{2}B & \ \ 29\ \ 15 \cdot 5 \end{array}$$

	° ′ ″		
½ B	29 15·5	cos	9·940763
			2
			19·881526
	° ′		
AB	40 37	sin	9·813578
BC	91 3	sin	9·999927
	2)131 40		2)19·695031
½ (AB + BC) -	65 50	sin	9·847515
θ -	44 44		
{½ (AB + BC) + θ}	110 34	sin	9·971398
{½ (AB + BC) − θ}	21 6	sin	9·556299
	° ′		2)19·527697
	35 29	sin	9·763848
	2		
AC	70 58		

CHAPTER V.

MATHEMATICAL ASTRONOMY.

Having referred to most of the circles of the heavens in Part I., Chap. 20, it is not necessary to mention them again here, but we will now speak of the mathematical principles of the science, computations of latitude and longitude, and the investigations of the rules by which the latter are found, and which are based entirely on the trigonometrical formulæ already proved.

As the computation of latitude and longitude depends mostly upon the altitudes of celestial bodies, we will speak of that in the first place. We have already said that the altitude of a heavenly body is its height above the horizon, or more accurately, the arc of a circle of altitude intercepted between the object and the horizon, the eye of the observer being supposed to be placed at the centre of the Earth. It is evident that, as altitudes are observed on the surface of the Earth, they require several corrections before the true altitudes can be deduced from them. These corrections are for semi-diameter, refraction, dip, and parallax. By the semi-diameter of a celestial object is meant the angle which the radius of its apparent circular disc, subtends at the eye of the observer. When the object is in the horizon, the semi-diameter and parallax are called the horizontal semi-diameter and parallax, and the parallax at any altitude is called the parallax in altitude.

The Sun's semi-diameter is given in the Nautical Almanack for every day of the month. His mean horizontal parallax is about 8·75″, and it never differs more from that quantity than about a quarter of a second; and his semi-diameter varies from 15 to 16 minutes. The parallax of the Sun and planets is so small that this correction with respect to them is too minute to be observed. The Moon's parallax is to its semi-diameter always very nearly as 11 to 3. When the eye is above the surface of the Earth, the visible horizon appears depressed, as the line from the eye touching the horizon is a tangent to the Earth at the point of contact. The depression of this line below the horizontal plane passing through the eye of the observer is called the dip, and the altitude is too great by this depression. The correction for dip is therefore subtractive.

When a ray of light passes from a rarer to a denser medium, it is bent towards the perpendicular, at the point at which it entered. This bending is called refraction. Now the strata of the atmosphere through which the rays of light pass to the Earth increase in density as they approach the surface, being compressed by the weight of the superincumbent atmosphere; and therefore the rays of light from celestial objects are bent downwards as they approach the Earth, and consequently to us all celestial objects appear more elevated than they would be, or their altitudes are greater than they would be if light were not refracted.

With respect to the semi-diameter of the object, as it is the altitude of the centre that is required, the semi-diameter must be added to the observed altitude when the lower edge or limb is observed, and subtracted from it when the upper limb is observed. The Moon's semi-diameter is sensibly greater when she is viewed from the Earth's surface, than if it were seen from its centre, which is that given in the Nautical Almanack.

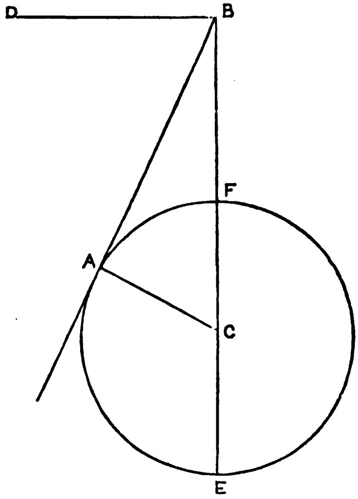
Altitudes are sometimes observed on land by reflection, from what is termed an artificial horizon, and which is merely the horizontal surface of a fluid.

Correction for Dip.

Let the circle represent the Earth, B the position of the observer, then BF is the height of the eye above the surface. Let BA be a tangent to the surface at the point A, and BD a horizontal line.

Let BF $= h$, CF the radius of the Earth $= r$, and the angle DBA is the dip.

Fig. 26.



$$\text{Since BAC} = 90^\circ, \text{ therefore ABC} + \text{ACB} = 90^\circ.$$
$$\text{But DBC} = 90^\circ.$$
$$\text{Therefore DBC} = \text{ABC} + \text{ACB}.$$

Omit the angle ABC.

$$\text{Then DBA} = \text{ACB},$$
$$\text{or Dip} = \text{ACB}.$$
$$\text{Cos ACB or Dip} = \frac{\text{AC}}{\text{CB}} = \frac{r}{r+h}.$$
$$\text{Then } 1 - \cos \text{Dip} = 1 - \frac{r}{r+h} = \frac{h}{r+h} = \frac{h}{r} \text{ nearly.}$$

$$\text{or } 2 \sin^2 \tfrac{1}{2} \text{ dip} = \frac{h}{r}$$

$$\text{or } \sin^2 \tfrac{1}{2} \text{ dip} = \frac{h}{2r}.$$

$$\text{But } \sin \tfrac{1}{2} \text{ dip} = \tfrac{1}{2} \text{ dip}'' \times \sin 1''.$$

$$\text{Therefore } \tfrac{1}{2} \text{ Dip}'' \times \sin 1'' = \sqrt{\frac{h}{2r}} = \sqrt{h} \times \sqrt{\frac{1}{2r}}$$

$$\text{But } \sqrt{\frac{1}{2r}} \text{ being small may be considered 0.}$$

$$\text{Therefore } \tfrac{1}{2} \text{ Dip}'' = \sqrt{h} \times \frac{1}{\sin 1''},$$

$$\text{or Dip} = \sqrt{h} \times \frac{2}{\sin 1''}$$

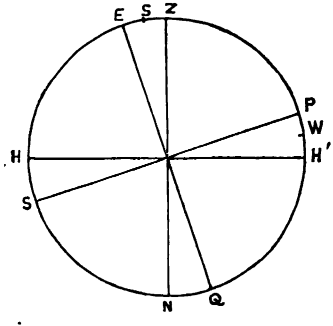
$$\text{or Dip} = \sqrt{h} \text{ nearly.}$$

CHAPTER VI.

Problem I.

To find the Latitude from the Altitude of an Object when on the Meridian.

Fig. 25.



Let the circle represent the celestial meridian, then Z is the zenith, P is the elevated pole, PS the polar axis of the Earth produced, EQ the plane of the equator, HH' that of the horizon, N the nadir. Then, since the height of a person above the terrestrial equator is his latitude, ZE, the height of his zenith above the celestial equator, measures the latitude.

To prove that the height of the pole above the horizon measures the latitude.

$$\begin{aligned} \text{Since } PE &= 90^\circ \\ \text{and } ZH' &= 90^\circ; \\ \text{therefore } PE &= ZH' \\ \text{or } EZ + ZP &= ZP + PH'. \end{aligned}$$

Omit ZP;

$$\text{then } EZ = PH';$$

or PH′ measures the latitude.

Then suppose the Sun to be at S, it is then above pole.

Then ZS is his zenith distance.
PS „ polar distance.
SE „ declination.
SH „ altitude.

$$\text{Now } ZE = ZS + SE,$$

$$\text{or, lat.} = \text{zen. dist.} + \text{decln.},$$

when the object is above pole.

Secondly, suppose the object to be below pole, as W.

$$\text{Then } PH' = PW + WH'$$

$$\text{or lat.} = \text{polar dist.} + \text{alt.}$$

From this we see that when the object is above pole, the difference between the zenith distance and declination must be taken; but when below pole, the sum of the polar distance and altitude must be taken, to find the latitude.

To find the Greenwich Date from that at the Place of Observation.

Since the Sun passes over 15° in one hour, we find that in order to convert longitude into time we must divide by 15. For instance, if we wish to express 30° of longitude in time, we get $\frac{30^\circ}{15^\circ} = 2$ hours. As the Earth turns on its axis from west to east, it is evident that the Sun will appear to come to a place east of Greenwich, sooner than to one west. The Greenwich time will therefore be earlier than that in east longitude, and later than that in west longitude.

From this arises the following rule:—

If the place be east of Greenwich, subtract the longitude in time, in order to find the Greenwich time; if west, add it.

If we wish to convert time into longitude we must multiply by 15. When the longitude is given in degrees, minutes, and seconds, it will be found easier to multiply by 4 and divide by 60, than to divide by 15, and the reverse in converting time into longitude.

Examples.

On March the 5th I took an observation at $5^h\ 30^m$ p.m. in longitude 25° 30′ 10″ W.; required the Greenwich date.

	° ′ ″
Longitude	25. 30. 10 W.
	4
	60)102. 00. 40
Longitude in time .	$1^h 42^m 00^s{\cdot}7$

	D. H. M. S.
Date at place March	5. 5. 30. 00
Longitude in time : +	1. 42. 1
Greenwich date .	5. 7. 12. 1

On May 24th, at $6^h\ 30^m$ a.m., I took an observation in longitude 25° 30′ 10″ E., required the Greenwich date.

As before, 25° 30′ 10″ = $1^h\ 42^m\ 1^s$

and date at place astronomically expressed,

	D. H. M. S.
May	23. 18. 30. 00
Longitude . . —	1. 42. 1
Greenwich date	23. 16. 47. 59 (astronomically),

or May $24^d\ 4^h\ 48^m$ a.m., civil date.

Having shown how to find the Greenwich date, the following steps may be put down as the principal parts of the problem.

1. Find the Greenwich date.
2. For the above date take out the declination from the Nautical Almanac, and if the object be the Sun, take out the semi-diameter also (page 1 of the month).
3. Correct the altitude for index error, dip, &c.

4. Find the latitude by the rule already investigated.

In the following examples the declination and true altitude are given instead of referring to the Nautical Almanac, and to a dip and refraction table.

1. If on May 4th, in longitude 31° W., the observed meridian altitude of the Sun's lower limb be 56° 20′ 30″ (zenith distance south), and height of the eye 20 feet; required the latitude.

	° ′ ″
Declination on May 4th	**16. 1. 39 N.**
Semi-diameter . . .	**15. 52**
Dip	**4. 24**
Refraction	**0. 38**
Parallax	**0. 5**

Greenwich Date.

		D. H. M. S.
Time at place (astronomically) when the sun passed the meridian . .	**May**	**4. 0. 0. 0**
Longitude in time	**„**	**2. 4. 0**
Greenwich date		4. 2. 4. 0

	° ′ ″	
Observed altitude	56. 20. 30	
Dip	− 4. 24	
	56. 16. 6	
Refraction . . .	− 38	
	56. 15. 28	
Semi-diameter .	+ 15. 52	
	56. 31. 20	
Parallax . . .	+ 5	
True altitude . .	56. 31. 25	
	90	
Zenith distance .	33. 28. 35	reckoned S.
Declination . .	16. 1. 39	N.
Latitude . . .	17. 26. 56	S.

Note.—If the zenith distance and declination be of like denominations, take the sum for the latitude, if not, take the difference.

2. If on July 4th the observed altitude of the Sun's lower limb on the meridian below pole be 6° 26′, in longitude 21° W., height of the eye 18 feet; required the latitude.

Corrected declination 22° 53′ 19″ N., semi-diameter 15′ 46″, dip 4′ 11″, refraction 7′ 57″, parallax 9″.

Greenwich date = July 4^d 13^h 24^m (12^h being added to the time of transit, the object being below pole.)

	° ′ ″
Observed altitude	6. 26. 0
Dip	— 4. 11
	6. 21. 49
Refraction . . .	— 7. 57
	6. 13. 52
Semi-diameter .	+ 15. 46
	6. 29. 38
Parallax	+ 9
True altitude . .	6. 29. 47

	° ′ ″		° ′ ″
Declination	22. 53. 9	True altitude.	6. 29. 47
	90	Co-declination	67. 6. 51
Co-declination	67. 6. 51	Latitude . .	73. 36. 38 N.

The latitude is of the same name as the declination.

In finding the latitude from the meridian altitude of a star, the same course is followed, excepting that the semi-diameter and parallax are so small as to be rendered unimportant.

CHAPTER VII.

Problem II.

TO FIND THE LATITUDE FROM AN ALTITUDE OF THE POLE-STAR.

The Pole-star is an object which describes a circle round the pole in 24 hours. If we imagine the whole celestial sphere to be moving round on its axis—that of the Earth produced—then the nearer we go towards the pole the smaller will the circles be which the bodies describe. The distance of this star from the pole is so small that we may consider the circle which it describes as a plane, of which the pole is the centre, and the star's polar distance the radius.

If it were placed immoveable at the pole, its altitude would be the latitude (*see* Chap. VI.), but as it is not so there are certain corrections to be applied, either + or —, according to the position of the star, before we can deduce the altitude of the pole from that of the star.

When the star is on the meridian above pole, its altitude exceeds that of the pole by its polar distance. When below pole, its altitude is less than that of the pole by the same quantity.

When six hours distant from the meridian its altitude is nearly equal to that of the pole, and, in rough calculations, may be taken for it.

In all other positions its polar distance may be considered as the hypothenuse of a right-angled triangle, of which the perpendicular is the difference between its altitude and that of the pole; and the acute angle included between those sides is the star's least meridian distance.

The position of the Pole-star is found from two stars of the Great Bear, which, if joined, the line produced will nearly pass through the Pole-star; these two stars are called the Pointers.

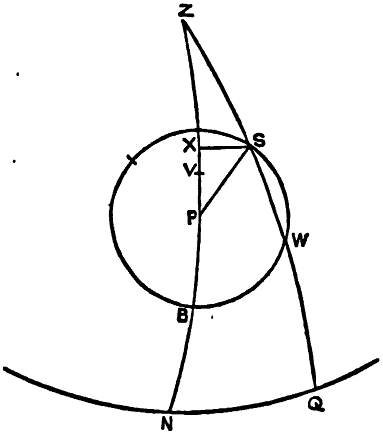
Investigation of the Problem.

Let Z be the zenith, P the pole, SWB the diurnal circle described by the Pole-star.

Let S be the position of the star when observed, ZQ a circle of altitude passing through the star.

Then ZS = zenith distance of the star, PS = polar distance, and angle XPS = meridian distance of star, or hour angle = h.

Fig. 26.



Draw SX perpendicular to ZN.

Then since the angle ZXS = 90°, therefore ZSX $<$ 90°.

Therefore ZS $>$ ZX;

and, therefore, a circle described with centre Z, and radius ZS will cut ZN below X. Let it cut in V.

Then PX is the first correction } to be applied to the
VX is the second } altitude.

The triangle PXS, being small, may be considered as a plane triangle, right-angled at X.

$$\text{Then PX} = \text{PS} \cdot \cos \text{SPX};$$

or, 1st correction $= p \cdot \cos h$.

$$\text{Again,} \quad \text{ZS} = \text{ZV} = (\text{ZX} + \text{XV})$$

$$\text{and } \cos \text{ZS} = \cos (\text{ZX} + \text{XV})$$

$$= \cos \text{ZX} \cdot \cos \text{XV} - \sin \text{ZX} \cdot \sin \text{XV}.$$

And, by Napier's rules (spherical trigonometry), since ZXS = 90°, $\cos \text{ZS} = \cos \text{ZX} \cdot \cos \text{SX}$;

$$\text{then } \cos \text{ZX} \cdot \cos \text{SX} = \cos \text{ZX} \cdot \cos \text{XV} - \sin \text{ZX} \sin \text{XV};$$

but, as VX is small, $\cos \text{VX} = 1$ nearly.

Therefore $\cos ZX \,.\, \cos SX = \cos ZX - \sin ZX \,.\, \sin VX$.

And, by transposition,

$$\begin{aligned} \text{Sin } ZX \,.\, \sin VX &= \cos ZX - \cos ZX \,.\, \cos XS. \\ &= \cos ZX\,(1 - \cos XS), \\ &= \cos ZX \,.\, 2 \sin^2 \frac{XS}{2}. \end{aligned}$$

and dividing by sin ZX,

$$\text{Sin } VX = \cot.\, ZX \,.\, 2 \sin^2 \frac{XS}{2}.$$

$$\left\{ \begin{array}{l} \text{But } \cot.\, ZX = \tan XN = \tan a \text{ nearly.} \\ \text{and } \sin VX = VX'' \,.\, \sin 1'' \\ \text{also, } \sin \dfrac{SX}{2} = \dfrac{XS''}{2} \,.\, \sin 1'' \\ \text{and } \sin^2 \dfrac{SX}{2} = \dfrac{SX^2}{4} \sin^2 1'' = \dfrac{(p \,.\, \sin h)^2}{4} \,.\, \sin^2 1' \\ \text{and } 2 \sin^2 \dfrac{SX}{2} = \dfrac{(p \,.\, \sin h)^2}{2} \sin^2 1''. \end{array} \right.$$

And, substituting these values,

$$VX'' \,.\, \sin 1'' = \tan a \,.\, \frac{(p.\, \sin h)^2}{2} \sin^2 1''.$$

Dividing by sin 1″,

$$\text{2nd cor.} = VX = \tfrac{1}{2} \tan a \,.\, (p \sin h)^2 \,.\, \sin 1''.$$

Example.

If, on October 7th, at $9^h\ 26^m$ p.m., in longitude 16° W., the observed altitude of the Pole-star be 46° 18′ 0″, height of the eye 13 feet, required the latitude.

	Greenwich Date.	Longitude.
	D. H. M. S.	
Date at place Oct.	7. 9. 26. 0	16° W.
Longitude +	1. 4. 0	4
	7. 10. 30. 0	6,0)6,4
		+ $1^h 4^m$

From the Nautical Almanac.

	H.	M.	S.
R. A. of the star =	1.	8.	9·22
	°	′	″
Declination . =	88.	33.	28·4
	90.	0.	0
	1.	26.	31·6
	60		
	86′		
	60		
p.	5191.6″		

Mean Sun's Right Ascension.

		H.	M.	S.
Sidereal time - -		13.	3.	22
Acceln. for G. T. - From Nautical Almanac	10h =		1.	38·6
	30m =			4·9
		13.	5.	5·5

The Hour angle = Mean T. at pl. + Mean ☉'s R. A. − ✱'s R. A.

	H.	M.	S.
Mean time	9.	26.	0
Mean ☉s R. A.	13.	5.	5·5
	22.	31.	5·5
Star's R. A.	1.	8.	9·2
	21.	22.	56·3
	24.	0.	0
	2.	37.	3·7
	60		
4)	157.	3.	42
h	39.	15.	55

$$
\begin{array}{rl}
1\text{st Cor.} & = p\,.\cos h \\
p\ 5192 & = 3{\cdot}715335 \\
\cos h & = 9{\cdot}888858 \\
\hline
6{,}0)402{,}9. \quad \ldots & \ 3{\cdot}604193 \\
6{,}0)6{,}7.9 & \\
-\ 1.7.9 &
\end{array}
$$

$$
\begin{array}{lrrr}
 & {}^{\circ} & ' & '' \\
\text{Obs. alt.} & 46. & 18. & 0 \\
\text{Dip} & - & 3. & 33 \\
\hline
 & 46. & 14. & 27 \\
\text{Refn.} & - & & 55 \\
\hline
\text{True alt.} & 46. & 13. & 32
\end{array}
$$

$$2\text{nd Cor.} = \tfrac{1}{2}\tan a\,(p\,.\sin h)^2\,.\sin 1''$$

$$
\begin{array}{rl}
p\ \ 5192 & = 3{\cdot}715335 \\
\text{Sin } h & = 9{\cdot}801356 \\
\hline
 & \ 3{\cdot}516691 \\
 & \ \phantom{3{\cdot}51669}2 \\
\hline
(p\,.\sin h)^2 & \ 7{\cdot}033382
\end{array}
$$

$$
\begin{array}{rl}
(p\,.\sin h)^2 & = 7{\cdot}033382 \\
\tan a & = 10{\cdot}019714 \\
\sin 1'' & = 4{\cdot}685575 \\
\hline
2)54{\cdot}8 & \ 1{\cdot}731671 \\
+\ 27''.4 &
\end{array}
$$

The second correction is *always* positive. The first is positive or negative, according to the following :—

If the hour-angle be greater than 6^h and less than 18^h, it is +

If between 18^h and 24^h, it is −

If between 0^h and 6^h, it is −

In this example it is negative, therefore,

	°	′	″
True Alt. =	46.	13.	32
1st Cor. −	1.	7.	9
	45.	6.	23
2nd Cor. +			27
Latitude	45.	6.	50 N.

The value sin 1″ has a constant logarithm − 4·685575.

CHAPTER VIII.

Problem III.

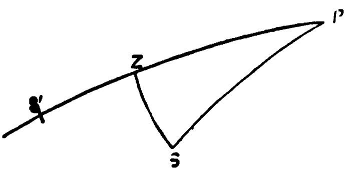
TO FIND THE LATITUDE FROM AN ALTITUDE OF THE SUN NEAR THE MERIDIAN.

In this problem an altitude is taken near noon, *i.e.*, either after 11 a.m. or before 1 p.m. The problem is generally known as "reduction to the meridian," which name has been given from the following :—

If the Sun were *on* the meridian, the problem would be reduced to that in Chapter VI., and for a very rough calculation, an altitude taken as near noon as possible might be considered a meridian altitude. But when the latitude is required to the nearest minute, a correction must be applied to the altitude, the formula for finding which is now to be investigated.

Let Z be the zenith, P the pole, S the position of the Sun near the meridian when observed. Let S′ be his position on the meridian, on the supposition that his polar distance does not change after being observed at S.

Fig. 27.



Let PS = p, ZP = l', ZS = z, and ZS′ = z', and the angle SPZ = h.

By supposition, $PS = PS'$

$$\text{or } p = z' + l' ;$$
$$\text{and } z' = p - l'$$
$$\text{and } \cos z' = \cos (p - l') = \cos p . \cos l' + \sin p \sin l'.$$

And, since PZS is a spherical triangle,

$$\cos z = \cos p . \cos l' + \sin p . \sin l' \cos h.$$

Then $\cos z' - \cos z = \sin p . \sin l' (1 - \cos h)$,

$$\text{or, } 2 \sin \frac{z + z'}{2} . \sin \frac{z - z'}{2} = \sin p . \sin l' . 2 \sin^2 \tfrac{1}{2} h ;$$

$$\text{but } \frac{z + z'}{2} = z' \text{ nearly,}$$

$$\text{and } \frac{z - z'}{2} = \frac{r}{2} \text{ where } r = \text{reduction.}$$

Therefore substituting

$$2 \sin z' . \sin \frac{r}{2} = \sin p . \sin l' . 2 \sin^2 \tfrac{1}{2} h ;$$

$$\text{but } \sin \frac{r}{2} = \frac{r''}{2} . \sin 1''.$$

$$\text{therefore } 2 \sin \frac{r}{2} = r'' \sin 1''$$

Then $\sin z' . r'' \sin 1'' = \sin p . \sin l' . 2 \sin^2 \frac{1}{2} h$;

$$\text{or } r'' = \sin p . \sin l' \operatorname{cosec} z' . \frac{2 \sin^2 \frac{1}{2} h}{\sin 1''}.$$

$$r'' = \cos \text{dec} . \cos \text{lat} . \sec \text{mer alt} . \frac{2 \sin^2 \frac{1}{2} h}{\sin 1''}.$$

Example.

The true latitude is required to be found from the following :—

In latitude nearly 12° S. the observed altitude of the Sun's lower limb was 55° 38′ 56″, index error of sextant — 55″, dip 0, refraction 40″, semi-diameter 15′ 46″, parallax 5″, corrected declination 22° 5′ 40″ N., and value of 2 sin² ½ h 148.

1. To find the meridian zenith distance :—

	° ′
Latitude	12. 0 S.
Declination	22. 6 N.
M. Z. D.	34. 6

2. To find the reduction, formula :—

$$r'' = \cos \text{dec} \cos \text{lat} \,.\, \text{cosec MZD} \,.\, \log \frac{2 \sin^2 \frac{1}{2} h}{\sin 1''}.$$

$$\begin{aligned} \text{Cos dec} &= \cos 22^\circ.\ 6' = 9{\cdot}966859 \\ \text{Cos lat} &= \cos 12.\ 0 = 9{\cdot}990404 \\ \text{Cosec MZD} &= \text{cosec } 34.\ 6 = 10{\cdot}251317 \\ \log .\ \frac{2 \sin^2 \frac{1}{2} h}{\sin 1''} &= \log 148 = 2{\cdot}169968 \end{aligned}$$

6‸0) 23‸9. 1 = 2·378548

+ 3. 59. 1

3. To find the latitude :—

	° ′ ″
Observed altitude	55. 38. 56
Index error -	— 55
	55. 38. 1
Refraction -	— 40
	55. 37. 21
Semi-diameter -	+ 15. 46
	55. 53. 7
Parallax - -	+ 5
True altitude	55. 53. 12
Reduction -	+ 3. 59
Meridian altitude	55. 57. 11
	90. 0. 0
Zenith distance -	34. 2. 49 S.
Declination	22. 5. 40 N.
Latitude - -	11. 57. 9 S.

There are many more steps in this problem than those shown above, which render it much more complicated. There are two Greenwich times to be found, for one of which the declination is taken from the Nautical Almanac. The value of $\frac{2\sin^2\frac{1}{2}h}{\sin 1''}$ is to be obtained from a table of hour angles, and to obtain the hour angles a third date is found, called "chronometer at noon." I have omitted these steps, merely wishing to point out the application of the formula. In case the student should wish to know the complete work of any problem, I refer him to a work entirely devoted to nautical astronomy.*

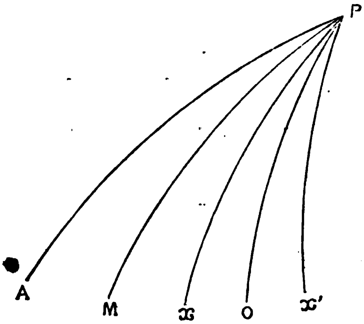
CHAPTER IX.

Problem IV.

TO FIND THE TIME AT A PLACE.

Let PA represent the meridian of the first point of Aries, PM that of the mean Sun, Px of any object x, and PO of the observer.

Fig. 28.



Then since right ascension is measured from A, we have,

$$APM + MPO = APO.$$

(1) Or mean time at place + mean Sun's RA = right ascension of meridian.

---

* The best perhaps is that by J. Riddle, Esq., of the Royal Naval Schools, Greenwich Hospital. Published by Law, Essex Street, Strand.

Again, since

$$x\text{PO} = \text{APO} - \text{AP}x, \text{ therefore,}$$

(2.) Westerly mer. dist. = R.A. of mer. − R.A. of object.

If the object be east of the meridian, as at x', its right ascension (APx') exceeds that of the meridian, and

$$\text{OP}x' = \text{AP}x' - \text{APO},$$

or easterly mer. dist. = object's R.A. − R.A. of mer.

Then the westerly meridian distance is,

$$24^{\text{h}} - \text{OP}x' = 24^{\text{h}} + \text{APO} - \text{AP}x',$$

and therefore it may be found by subtracting the right ascension of the object from that of the meridian increased by 24 hours.

Again,

$$x\text{PO} + x\text{PA} = \text{APO},$$

or R.A. of object + westerly mer. dist. of object = sidl. time
= R.A. of mer.,

$$\text{and MPO} + \text{MPA} = \text{APO}.$$

(3.) Or, mean time at pl. + mean sun's R.A. = R.A. of mer., or sidereal time.

$$\text{APO} - \text{APM} = \text{MPO}.$$

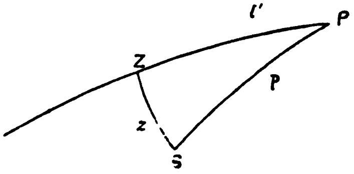
(4.) Or, R.A. of mer. − mean sun's R.A. = mean time, or westerly mer. distance.

The above rules serve for determining the meridian distance or hour angle of any object, the time being given, or the time when the meridian distance is known.

1. The westerly meridian distance of the true sun measures apparent time.
2. The westerly meridian distance of the mean sun measures the mean time.
3. The westerly meridian distance of the first point of Aries measures sidereal time.

Formula for computing Time.

Fig. 29.



Let z be the zenith, P the pole, S the object's position when observed.

Let P$z = l'$, PS $= p$, and ZS $= z$, and the angle zPS $= h$, or hour angle.

$$\text{Then } \cos h = \frac{\cos z - \cos l' \cos p}{\sin l' \sin p}.$$

$$= \frac{\sin a - \sin l \cos p}{\cos l \sin p}.$$

$$\text{and } 1 - \cos h = 1 - \frac{\sin a - \sin l \cos p}{\cos l \,.\, \sin p}.$$

$$= \frac{\cos l \sin p + \sin l \cos p - \sin a}{\cos l \sin p}$$

$$= \frac{\sin (p + l) - \sin a}{\cos l \sin p}$$

$$= \frac{2 \cos \frac{1}{2} (p + l + a) \sin \frac{1}{2} (p + l - a)}{\cos l \sin p}.$$

$$\text{Let } (p + l + a) = 2\,\mathrm{S};$$

$$\text{then } \tfrac{1}{2} (p + l - a) = \mathrm{S} - a,$$

$$\text{and } 1 - \cos h = 2 \sin^2 \tfrac{1}{2} h.$$

$$\text{Therefore } 2 \sin^2 \tfrac{1}{2} h = \frac{2 \cos \mathrm{S} \,.\, \sin (\mathrm{S} - a)}{\cos l \sin p}$$

$$\text{or } \sin \tfrac{1}{2} h = \sqrt{\sec l \operatorname{cosec} p \,.\, \cos \mathrm{S} \,.\, \sin (\mathrm{S} - a)}.$$

In this problem an example is given merely upon the application of the formula, supposing the polar distance (p) to have been already found. The equation of time which converts apparent into mean time, and *vice versâ*, is taken from the Nautical Almanac.

Example.

On December the 8th, at A.M. time, in lat. 32° 40′ 18″ S., when the sun's polar distance was 67° 20′ 56″, his corrected altitude was 22° 54′ 11″. Equation of time 8′ 15·8″ (to be subtracted from apparent time). Required the true mean time at the place of observation.

NOTE.—If the time be A.M. the sun is east of the meridian, and the angle found must be subtracted from 24 hours to get the westerly meridian distance or apparent time; but if the given time be P.M. the angle is already westerly. In either case the equation of time is applied; and the hour angle of any object is reckoned from the time it comes to the meridian till its next transit, which interval in the case of the sun is 24 hours.

Formula $\sin \frac{1}{2} h = \sqrt{\sec l . \mathrm{cosec}\, p . \cos S . \sin (S-a)}$

	° ′ ″		
a	22. 54. 11		
l	32. 40. 18	sec	10·074802
p	67. 20. 56	cosec	10·034861
	2)122. 55. 25		
S	61. 27. 42	cos.	9·679198
a	22. 54. 11		
(S − a)	38. 33. 31	sin	9·794708
			2)19·583569
	° ′ ″		
$\frac{1}{2} h$	38. 15. 10	sin	9·791784
	2		
h	76. 30. 20		
	4		
	6,0)30,6. 1. 20		
	5. 6. 1·3		
Subtr. from	24. 0. 0		
	18. 53. 58·7		
Equation of time	− 8. 15·8		
M. T. at place	18. 45. 42·9		

Or the observation was taken at 6h. 45m. 42s. A.M. civil date.

On February 10th the computed hour angle of the star Regulus was 49°. 2′. 18″. If the R.A. of Regulus be 10h. 0m. 39·7s., and that of the mean sun 21h. 21m. 49·4s. (the star being east of the meridian when observed); required the true mean time.

	°	′	″
Hour angle	49.	2.	18
			4
	6,0)19,6.	9.	12
Easterly hour angle	3.	16.	9·2
Subtract from	24.	0.	0
	20.	43.	50·8
R.A. of Regulus	10.	0.	39·7
R.A. of the meridian	30.	44.	30·5
R.A. of the mean Sun	21.	21.	49·4
Mean time - -	9.	22.	41·1 (Chap. IX., R. 4.)

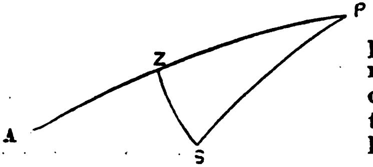
The time thus found is P.M., the object being a star.

CHAPTER X.

Problem V.

TO FIND THE AZIMUTH OF AN OBJECT.

Fig. 30.



Let Z be the zenith, P the pole. Then the angle AZS measures the azimuth, reckoned from the N. or S., in the converse way to the latitude.

$$\text{Cos PZS} = \frac{\cos p - \cos l' \cos z}{\sin l' \sin z}$$

$$\begin{aligned} \text{But } \cos \text{PZS} &= -\cos \text{AZS}, \\ \text{and } \cos l' &= \sin l\,; \\ \text{and } \cos z &= \sin a \\ \sin l' &= \cos l \\ \sin z &= \cos a. \end{aligned}$$

$$\text{Then} -\cos AZS = \frac{\cos p - \sin l \sin a}{\cos l . \cos a}$$

$$1 - \cos AZS = 1 + \frac{\cos p - \sin l \sin a}{\cos a \cos l}$$

$$= \frac{\cos a \cos l + \cos p - \sin a \sin l'}{\cos a \cos l}$$

$$= \frac{\cos (a + l) + \cos p}{\cos a \cos l}$$

$$= \frac{2 \cos \frac{1}{2} (a + l + p) \cos \frac{1}{2} (a + l - p)}{\cos a \cos l}$$

Then if $a + l + p = 2\,S$

$$2 \sin^2 \frac{AZS}{2} = \frac{2 \cos S . \cos (S - p)}{\cos a \cos l}.$$

$$\text{Or } \sin \frac{AZS}{2} = \sqrt{\sec a \sec l . \cos S . \cos (S - p)}.$$

Example.

Given the true altitude of the Sun 28° 16′, the latitude 66° 12′ S., and the polar distance 81°, to find the true azimuth, the time of observation being A.M.

	° ′		
a	28. 16	sec	10·055146
l	66. 12	sec	10·394108
p	81. 0		
	2)175. 28		
S	87. 44	cos	8·597152
S − p	6. 44	cos	9·996994
			2)19·043400
	° ′		
½ AZS	19. 25	sin	9·521700
	2		

True azimuth N 38. 50 E., as the altitude is increasing.

Problem VI.

2. WHEN THE ALTITUDE IS NOT TAKEN.

In this case (*see* last fig.) SP, PZ, and angle P are given, from which, by Napier's Analogies, we find the angle Z.

Given, the Sun's hour angle 64° 35′, the declination 2° S., and the latitude 64° 10′ S., to find the azimuth.

$$\text{Tan}\ \tfrac{1}{2}(Z+S)=\cos\tfrac{1}{2}(p-l')\sec\tfrac{1}{2}(p+l')\cot\frac{P}{2}$$

$$\text{Tan}\ \tfrac{1}{2}(Z-S)=\sin\tfrac{1}{2}(p-l')\,\text{cosec}\,\tfrac{1}{2}(p+l')\cot\frac{P}{2}.$$

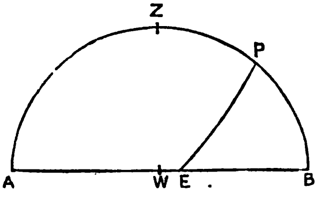
	° ′				
$\frac{1}{2}P =$	32. 18	cot	10·199164	cot	10·199164
$p =$	88. 00				
$l' =$	25. 50				
	2)62. 10				
$\frac{1}{2}(p-l')$	31. 5	cos	9·932685	sin	9·712889
$\frac{1}{2}(p+l')$	56. 55	sec	10·262920	cosec	10·076819
$\frac{1}{2}(Z+S)$	68. 3	tan	10·394769		
$\frac{1}{2}(Z-S)$	44. 16	. . . . . .		tan	9·988872
	112. 19	P Z S (*see* fig.)			
	180. 0				
Azimuth	67. 41	A Z S (*see* fig.)			

Reckoned N E.

Problem VII.

TO FIND THE AMPLITUDE OF AN OBJECT.

Fig. 31.



The amplitude of an object (*see* Part I., last Chapter), is measured from the east point when the object is rising, and from the west when it is setting.

In the case of the Sun, the amplitude is 0 on the days which mark the equinoxes.

It increases towards the north from March till June, decreases from June till September; increases southward from September till December, and decreases from December till March. On the days of the equinoxes, the Sun travels on the equator, and travels on a parallel of north declination from March till September, and again till March on a parallel of south declination. It is evident, therefore, that the higher his declination, the greater will be his amplitude north or south. This will be shown from the following formula.

In the figure, let Z be the zenith; P, the pole; AB, the horizon; W, the east or west point. Then, if the object be at E when rising, EW is its amplitude.

And in the right-angled triangle EBP, by Napier's rules:—

$$\text{Cos EP} = \cos \text{EB} \cos \text{PB};$$

$$\text{or, } \cos \text{EP} = \sin \text{EW} \,.\, \cos \text{PB};$$

$$\text{and } \sin \text{EW} = \cos \text{EP} \,.\, \sec \text{PB},$$

$$\textit{i.e.,} \ \sin \text{amp.} = \sin \text{dec.} \sec \text{lat.}$$

Now, since the latitude may be considered as constant, sin amp. varies as sin dec., or the amplitude varies as the declination.

Example.

In latitude 25°. 30′ S, if the declination of the Sun be 24° 20′ N, required the rising amplitude.

$$\text{Sin amp} = \sin \text{dec.} \sec \text{lat.}$$

$$\begin{array}{lr}
 & {}^{\circ} \quad {}' \qquad\qquad \\
\sin \text{dec.} = \sin\ 24.\ 20 = & 9{\cdot}614944 \\
\sec \text{lat.} = \sec\ 25.\ 30 = & \underline{10{\cdot}044512} \\
\text{E } 27^{\circ}.\,9'\text{N} \,.\ \sin & \underline{\underline{9{\cdot}659456}}
\end{array}$$

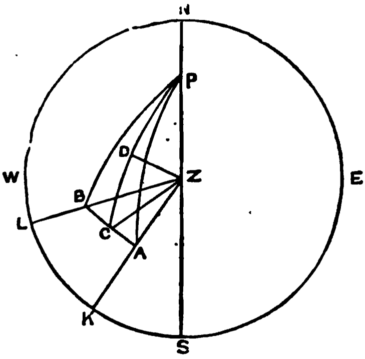
The amplitude is reckoned contrary to the latitude, and E since the object is rising.

CHAPTER XI.

Problem VIII.

OF FINDING THE LATITUDE BY IVORY'S METHOD.

Fig. 32.



Two observations are taken, either both a.m. or p.m., or one a.m. and the other p.m. In the figure, the two positions of the object are A and B, both on the same side of the meridian N S. Z L and Z K are circles of altitude passing through the two positions of the object.

Then AL and BK are the altitudes ; ZA and ZB, the zenith distances ; PA and PB, the polar distances; and the angle BPA measures the elapsed time between the two observations. Bisect the angle BPA by PC. Then, if the polar distances PA and PB are supposed equal, AB will be bisected in C. From Z, draw ZD perpendicular to PC.

Let the equal polar distances $= p$, the angle APC $= h$, half the sum and half the difference of the altitudes $=$ S and D respectively. Let the arcs AC, CP, ZD, CD, and DP be represented by (A), (E), (I), (O), and (E — O).

1. In the triangle APC by Napier's rules.

$$\text{Sin AC} = \sin \text{AP} \sin \text{APC},$$
$$\text{or, } \sin (\text{A}) = \sin p \sin h.$$

2. $$\text{Cos AP} = \cos \text{AC} \,.\, \cos \text{CP}.$$
$$\text{Therefore } \cos \text{CP} = \cos \text{AP} \sec \text{AC},$$
$$\text{or } \cos \text{E} = \cos p \,.\, \sec (\text{A}).$$

3. Since PCA and PCB are right angles,

$$\text{Cos ZCA} = \sin \text{ZCD and } \cos \text{ZCB} = - \sin \text{ZCD}.$$

$$\text{Sin AK} = \cos \text{ZA} = \cos \text{ZC} \,.\, \cos \text{CA} + \sin \text{ZC} \,.\, \sin \text{CA} \sin \text{ZCD}.$$
$$\text{Sin BL} = \cos \text{ZB} = \cos \text{ZC} \,.\, \cos \text{CB} - \sin \text{ZC} \,.\, \sin \text{CB} \sin \text{ZCD}.$$

Therefore, by subtraction, observing that $CB = CA$,

$$\text{Sin } AK - \sin BL = 2 \sin ZC \,.\, \sin CA \,.\, \sin ZCD.$$

But $\sin ZC \sin ZCD = \sin ZD$.

Therefore $\sin AK - \sin BL = 2 \sin CA \sin ZD$,

$$\text{or, } \frac{2 \cos \frac{1}{2}(AK + BL) \sin \frac{1}{2}(AK - BL)}{2 \sin CA} = \sin ZD.$$

or, $\sin ZD = \cos S \,.\, \sin D \operatorname{cosec} A$.

i.e., $\sin (I) = \cos S \,.\, \sin D \operatorname{cosec} A$.

Again, by addition,

$$\text{Sin } AK + \sin BL = 2 \cos ZC \,.\, \cos CA.$$

But $\cos ZC = \cos ZD \cos DC$.

Therefore $\sin AK + \sin BL = 2 \cos ZD \cos DC \cos CA$.

$$\text{and } \frac{\sin AK + \sin BL}{2 \cos ZD \,.\, \cos CA} = \cos CD;$$

$$\text{or, } \frac{2 \sin \frac{1}{2} S \,.\, \cos \frac{1}{2} D}{2 \cos (I) \cos (A)} = \cos CD;$$

or, $\sin \frac{1}{2} S \,.\, \cos \frac{1}{2} D \,.\, \sec (A) \sec (I) = \cos (O)$.

Again, $PD = PC - CD = (E - O)$,

and $\cos PZ = \cos PD \cos DZ$;

i.e., $\sin \text{lat.} = \cos I \,.\, \cos (E - O)$.

Example.

On March 9th two altitudes of the Sun were taken in the southern hemisphere — 21°. 47′. 5″ and 38°. 42′. 38″. The true declination was 4°. 13′. 51″ S., and the time elapsed between the observations was $3^h. 45^m. 8^s.$; required the latitude.

	° ′ ″		° ′ ″	Elapsed time.
				h m s
1st alt.	21. 47. 5	Decln.	4. 13. 51 S	
2nd	38. 42. 38		90. 0. 0	2) 3. 45. 8
	2) 60. 29. 43	*p.*	85. 46. 9	*h* 1. 52. 34
				60
S.	30. 14. 51			
				4) 112. 34. 0
D.	8. 27. 46			*h* 28°. 8′. 30″

To find the latitude:—

$$\text{Sin A} = \sin p \sin h.$$

$$\begin{array}{rcrcl}
 & & {}^\circ \quad {}' \quad {}'' & & \\
\text{Sin } p &=& \sin 85.\ 46.\ \ 9 &=& 9\cdot998815 \\
\text{Sin } h &=& \sin 28.\ \ 8.\ 30 &=& 9\cdot673635 \\
\hline
 & & \text{(A) } 28.\ \ 3.\ 33 & \sin & 9\cdot672450
\end{array}$$

$$\text{Cos (E)} = \cos p \sec \text{(A)}.$$

$$\begin{array}{rcrcl}
 & & {}^\circ \quad {}' \quad {}'' & & \\
\text{Cos } p &=& \cos 85.\ 46.\ \ 9 &=& 8\cdot867909 \\
\text{Sec (A)} &=& \sec 28.\ \ 3.\ 33 &=& 10\cdot054304 \\
\hline
 & & \text{(E) } 85.\ 12.\ 16 & \cos & 8\cdot922213
\end{array}$$

$$\text{Sin (I)} = \text{cosec (A)} \cos \text{S} \,.\, \sin \text{D}.$$

$$\begin{array}{rcrcl}
 & & {}^\circ \quad {}' \quad {}'' & & \\
\text{Cosec (A)} &=& \text{cosec } 28.\ \ 3.\ 33 &=& 10\cdot327550 \\
\text{Cos S} &=& \cos 30.\ 14.\ 51 &=& 9\cdot936442 \\
\text{Sin D} &=& \sin \ \ 8.\ 27.\ 46 &=& 9\cdot167810 \\
\hline
 & & \text{(I) } 15.\ 40.\ 50 & \sin & 9\cdot431802
\end{array}$$

$$\text{Cos (O)} = \text{Sin S} \cos \text{D} \sec \text{(A)} \sec \text{(I)}.$$

$$\begin{array}{rcrcl}
 & & {}^\circ \quad {}' \quad {}'' & & \\
\text{Sin S} &=& \sin 30.\ 14.\ 51 &=& 9\cdot702203 \\
\text{Cos D} &=& \cos \ \ 8.\ 27.\ 46 &=& 9\cdot995245 \\
\text{Sec (A)} &=& \sec 28.\ \ 3.\ 33 &=& 10\cdot054304 \\
\text{Sec (I)} &=& \sec 15.\ 40.\ 50 &=& 10\cdot016471 \\
\hline
 & & \text{(O) } 54.\ \ 5.\ 43 & \cos & 9\cdot768223
\end{array}$$

$$\text{(E} - \text{O)}.$$

$$\begin{array}{rcr}
 & & {}^\circ \quad {}' \quad {}'' \\
\text{(E)} &=& 85.\ 12.\ 16 \\
\text{(O)} &=& 54.\ \ 5.\ 43 \\
\hline
\text{(E} - \text{O)} & & 31.\ \ 6.\ 33
\end{array}$$

$$\text{Sin lat.} = \cos \text{I} \,.\, \cos (\text{E} - \text{O}).$$

$$\begin{array}{rlcl}
 & \quad\;\; {}^{\circ} \quad\; {}' \quad\; {}'' & & \\
\text{Cos I} = & \cos 15.\ 40.\ 50 & = & 9{\cdot}983529 \\
\text{Cos (E} - \text{O)} = & \cos 31.\ \ 6.\ 33 & = & \underline{9{\cdot}932567} \\
 & \text{Latitude } 55.\ 31.\ 11 & \sin & \underline{\underline{9{\cdot}916096}}
\end{array}$$

* * *

CHAPTER XII.

Problem IX.

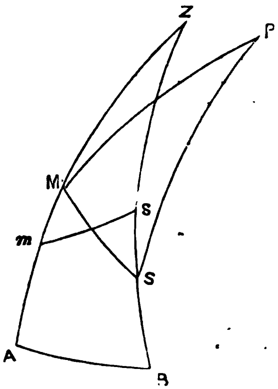
TO FIND THE LONGITUDE BY A LUNAR OBSERVATION.

Formula for clearing the Distance between the Sun and Moon.

In altitudes used only for computing the true distance, no great precision is necessary, but an altitude for computing the time, ought to be taken as exactly as possible. The greatest care, however, is required in measuring the distance with a sextant, as an error of one minute in it, will generally produce one of about two minutes of time, or half a degree in the longitude deduced from it. As the bright limb of the Moon is always turned towards the Sun, the distance of the nearest limbs of the Sun and Moon is always measured, and the distance of their centres is found by adding the semi-diameters of each. When the distance between the Moon and a star is taken, the semi-diameter of the Moon can only be added. But if the distance between the star and farther limb of the Moon be observed then the semi-diameter must be subtracted.

The distance thus obtained is called the apparent distance, and the true one is found from the formula here investigated.

Fig. 33.



In the figure,—

Let Z be the zenith, P the pole, M the true place of the Moon, S that of the Sun. Let *m* be the apparent place of the Moon, *s* that of the Sun, MS the true distance, *ms* the apparent distance. Then because the parallax of the Moon is greater than the refraction, therefore M is above *m*, and *s* above S.

In the investigation,—

Let m = apparent alt. of the Moon s = apparent alt. of the Sun.

M = true alt. of the Moon, S = true alt. of the Sun.

Then in the spherical triangle *mZs*, *mZ*, *sZ*, and *ms*, being known,

$$\text{Cos } Z = \frac{\cos ms - \cos mZ \cos Zs}{\sin m Z \,.\, \sin Zs}$$

Adding (1) to both sides and reducing, and observing that $m = (90^\circ - mZ)$ and $s = (90^\circ - sZ)$.

$$\text{Cos}^2 \frac{Z}{2} = \cos \tfrac{1}{2}\,(m + s + d) \cos \tfrac{1}{2}\,(m + s - d).\ \sec m \sec s.$$

$$\text{Let } X = \tfrac{1}{2}\,(m + s + d)\,;$$
$$\text{then } (X - d) = \tfrac{1}{2}\,(m + s - d),$$
$$\text{and } \cos^2 \tfrac{1}{2}\, Z = \cos X \,.\, \cos (X - d) \,.\, \sec m \,.\, \sec s.$$

Next in the triangle MZS, MZ, ZS, and angle Z now being known,

$$\text{Let } \sin^2 \theta = \sin MZ \,.\, \sin ZS \,.\, \cos^2 \tfrac{1}{2}\, Z\,;$$

$$\text{then } \sin^2 \frac{D}{2} = \sin (\tfrac{1}{2}\,(MZ + ZS) + \theta) \sin (\tfrac{1}{2}\,(MZ + ZS) - \theta)$$

Or, writing M and S for the true altitudes, which are the complements of the zenith distances MZ and SZ,

$$\text{Sin}^2 \theta = \cos M \,.\, \cos S \,.\, \cos X \cos (X - d) \,.\, \sec m \,.\, \sec s,$$

$$\text{and } \sin^2 \frac{D}{2} = \sin (\tfrac{1}{2}\,(MZ + ZS) + \theta) \sin (\tfrac{1}{2}\,(MZ + ZS) - \theta).$$

$$\text{But } ZM = (90^\circ - M),$$
$$\text{and } ZS = (90^\circ - S).$$
$$\text{Therefore } MZ + ZS = 180^\circ - (M + S),$$
$$\text{and } \tfrac{1}{2}\,(MZ + ZS) = 90^\circ - \tfrac{1}{2}\,(M + S).$$

This being substituted in the value of $\sin^2\frac{D}{2}$,

$$\sin^2\frac{D}{2}=\cos\left\{\frac{M+S}{2}+\theta\right\}\cos\left\{\frac{M+S}{2}-\theta\right\}$$

from which the true distance is found.

Example.

On October 11th, $23^h\ 26^m\ 20^s$ (Greenwich date), given the apparent distance (corrected for semi-diameter), 107° 52′ ; the apparent altitude of the Sun (s), 31° 11′; the apparent altitude of the Moon (m), 13° 29′ ; the true altitude of the Sun (S), 31° 9′ 33″; that of the Moon (M), 14° 18′ 5″ : required the true distance (D), and the longitude of the place of observation.

	° ′ ″		
d	107. 52. 0		
s	31. 11. 0	sec	10·012138
m	13. 29. 0	sec	10·067772
	2)152. 32. 0		
X	76. 16. 0	cos	9·375487
d	107. 52. 0		
$(d-X)$	31. 36. 0	cos	9·930300
	° ′ ″		
S	31. 9. 33	cos	9·932338
M	14. 18. 5	cos	9·986328
	2)45. 27. 38		2)19·304363
$\frac{1}{2}(M+S)$	22. 43. 49		
θ.	26. 40. 31	sin	9·652181
$\{\frac{1}{2}(M+S)+\theta\}$	49. 24. 20	cos	9·813381
$\{\frac{1}{2}(M+S)-\theta\}$	3. 56. 42	cos	9·998970
			2)19·812351
	° ′ ″		
	53. 40. 42	sin	9·906175

$$\begin{array}{rr} & {}^{\circ}\ \ {}'\ \ {}'' \\ \frac{1}{2}\,D = & 53.\ 40.\ 42 \\ & 2 \\ \hline D = & 107.\ 21.\ 24 \\ \hline\hline \end{array}$$

Hence the true distance is 107°. 21′. 24″.

The difference between this distance and one given in the Nautical Almanac, gives a certain number of degrees, minutes, and seconds, of which the proportional logarithm is taken, and another is given opposite the distance in the Nautical Almanac. The difference of the logarithms has in the table a certain number of hours, minutes, and seconds corresponding to it, which are those of the true Greenwich time. This is shown in the following.

To find the Longitude.

	° ′ ″		
True distance =	107. 21. 24	—	Proportional logarithm.
From Naut. Alm. at 21^h =	108. 28. 30	— 3397	From Naut. Alm.
because 21^h is the nearest to the number of hours in the Greenwich date of the question.	1. 7. 6	— 4289	From table of proportional logarithms.
		892	

	Corresponding hours, minutes, &c., from table of logarithms. h. m. s.
Difference, 892 - - - -	2. 26. 33
Number of hours for dist., from Naut. Alm.	21. 0. 0
True Greenwich date -	23. 26. 33

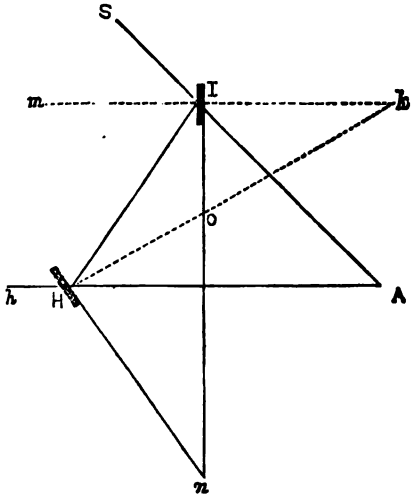
		d. h. m. s.
Greenwich date from distance =	Oct.	11. 23. 26. 33
Greenwich date from question =		11. 23. 26. 20
Difference - -		13″ W.

N.B.—W., because the first date is the greater.

CHAPTER XIII.

THE SEXTANT.

Fig. 34.



Let the line A *h* be directed towards the object *h*, and across the line let a mirror H be placed; in the figure the mirror may be supposed to stand perpendicular to the surface of the page.

Let KH be a perpendicular to the face of this mirror. Draw HI, making the angle IHK equal to the angle AHK. At I suppose another mirror to be placed with its plane perpendicular to the direction I *m*, and draw SI, making the angle SI *m* equal to the angle HI *m*.

If S represent another object, the ray SI will be reflected by the mirror I in the direction IH, and again by the mirror H in the direction HA. An eye at A, therefore, will see the image of S in the direction A *h*. The mirror H is half silvered, so that the object *h* may also be seen; then both *h* and the image of S will be seen in the section HA. Let SI be produced to meet HA in A, then the angle SA*h* is the angular distance at the eye between the objects S and *h*. By making the mirror I moveable about the centre, so that it will turn in the plane to which it is perpendicular, the images of objects at various angular distances may be made to coincide with *h*.

To prove that the inclination of the mirrors (the angle *n*) is equal to half the angle A.

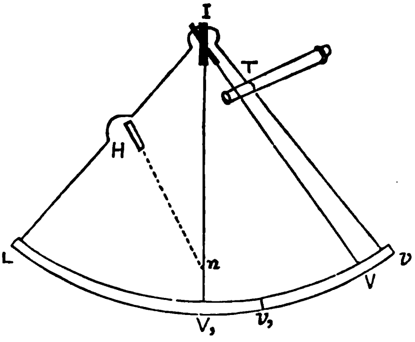
$$\angle \mathrm{I}o\mathrm{K} = \angle \mathrm{H}on \quad \text{(Euc. I., 15)}$$
$$\angle \mathrm{KI}o = \angle o\mathrm{H}n \quad \text{(each being } 90^\circ\text{)}.$$

Therefore $\angle n = \angle \mathrm{K}$.

$$\text{Again, } \angle\, SIH = \angle\, IHA + A \quad \text{(Euc. I., 32)},$$
$$\text{or } \tfrac{1}{2} \angle\, SIH = \tfrac{1}{2} \angle\, IHA + \tfrac{1}{2} A,$$
$$\text{or } \angle\, mIH = \angle\, IHK + \tfrac{1}{2} A.$$
$$\text{but } \angle\, mIH = \angle\, IHK + K.$$
$$\text{Therefore } IHK + K = \angle\, IHK + \tfrac{1}{2} A.$$
$$\text{Omit } \angle\, IHK.$$
$$\text{Then } K = \tfrac{1}{2} A.$$
$$\text{But } K = n.$$
$$\text{Therefore } n = \tfrac{1}{2} A.$$

The glasses are attached to a solid framework, and to the glass I is attached a moveable radius IV, carrying an index V, which moves over the arc or limb VL as I is turned about the centre. The limb VL is divided into degrees, &c., and these divisions should commence from the point at which the index stands when the mirrors I and H are parallel, for then their inclination is nothing, and the arc moved over by the index from that point will measure the inclination of the glasses, thus $VV_{,}$ measures $VIV_{,}$ or its equal, the angle n. As it is not the inclination of the glasses, but the angular distance of the two objects which is sought, the half degrees are marked as whole degrees on the divided limb, so that the degrees, &c. read off, are the measures of the angle subtended by the objects observed.

Fig. 35.



The radius $IV_{,}$ has been drawn in the same plane with the mirror I ; this is seldom the case in the sextant, and does not affect the principle, for if the radius were bent into the position Iv, the divisions would commence at v instead of V, and the index would arrive at v instead of $V_{,,}$ when carried along the limb, and $vv_{,} = VV_{,,}$ or the measured arc would remain the same.

If when the mirrors are parallel the index does not stand at zero, but rather in advance or to the left, then every arc read off will be too great; and, on the contrary, if it stand to the right, every measured arc will be too small; this constitutes the index error, which might be always ascertained by careful observations. The index at V is furnished with a scale for subdividing the divisions on the limb; this scale is called a Vernier from its inventor, and it is constructed thus:—Suppose, as is usually the case, the divisions on the limb to be each 10′, then 59 such divisions would contain 590′, and this length being divided into 60 equal parts, the value of the new divisions would be 9′ 50″, or 10″ less than the divisions on the limb. These 60 divisions constitute the vernier scale, which is attached to the extremity of the moveable radius IV, and slides along the limb LV.

It will be seen that 6 divisions of the vernier are just 1′ shorter than 6 divisions of the limb.

If, therefore, the stroke which marks the 6th division of the vernier, were to coincide with one of the strokes upon the limb, the index, or zero point of the vernier, would be just 1′ in advance of the corresponding mark or stroke upon the limb. In the same manner, if one of the marks which terminate the 12th, 18th, 24th, &c. divisions of the vernier, were thus to coincide with a mark on the limb, the index of the vernier would be 2′, 3′, 4′ in advance of the corresponding division of the limb, and thus the 10 minute divisions are divided into single minutes. These points of the vernier are distinguished by longer strokes to facilitate the counting.

I is called the index glass, H the horizon glass, LV the limb, and a telescope T is attached to the instrument, to enable the observer to mark the contact of the objects observed with greater accuracy. On shore an artificial horizon is used, consisting of a small rectangular trough, filled with liquid, generally mercury, and the angular distance between the object and its image in the mercury is observed, which is double the angle between the visual ray which comes from the object and the horizontal surface of the mercury, and therefore, double the altitude of the object.

The telescope is furnished with a screw to regulate its distance from the plane of the sextant, and to direct it more or less towards the silvered or unsilvered part of the horizon glass, and thus to render the object which is seen directly through the clear portion, and the image reflected from the silvered portion, equally distinct.

The adjustments of the instrument are, to make all the mirrors perpendicular, and the axis of the telescope parallel to the plane of the instrument. The frame which holds the index glass is fastened to the index by means of two screws behind it, and behind these is an adjusting screw. Having placed the index about the middle of the limb as V,I is placed, turn the face of the instrument upwards, and look obliquely into the glass; and if the image of VV, appears on a level with VV, itself, as seen by the eye, the index glass is perpendicular to the plane of the instrument; but if the image appears lower than VV, itself, tighten the adjusting screw, if higher slacken it, till the limb VV, and its image, seen by reflection in the glass, appear one continued plane.

To make the horizon glass perpendicular to the plane of the instrument. This adjustment is made in some instruments by means of a screw passing through the frame, and in others by a screw behind, turned by a key or a small capstan pin, which is put into a hole in the head of the screw.

When the instrument is furnished with a telescope, screw a dark glass on the end of the telescope, and looking at the Sun, make the two images pass over each other, and if they do not exactly cover in passing, turn the adjustment screw till they do so.

To make the axis of the telescope parallel to the plane of the instrument, turn the eye-piece of the telescope till two of the parallel wires in its focus appear parallel to the plane of the instrument, and bring the Sun and Moon in apparent contact on the wire next the instrument, then bring them instantly to the other wire, on which, if they still appear in contact, no adjustment is required. If they separate, slacken the screw farthest from the instrument in the ring which holds the telescope, and tighten the other; and *vice versâ* if they overlap. On a few repetitions this adjustment may be made perfect, and is not very liable to change.

To find the Index Error.

1. Move the index till the Sun or any distant object coincides with its image, and the distance of 0 on the index from 0 on the limb is the index error; subtractive when 0 on the index is to the left, and additive when it is to the right of 0 on the limb.

2. Or, make the images touch at the edges above and below, and when the readings are one right and the other left of zero on the limb, half their difference is the index error; but when they are both left or both right of zero, half their sum is the index error, subtractive when the greater reading is left, and additive when right of zero.

CHAPTER XIV.

THE TRANSIT INSTRUMENT.

A transit instrument is a telescope which revolves in the plane of the meridian on a horizontal axis, and has generally five wires in its focus, the middle one in the plane of the meridian, and the others parallel to, and equidistant from it; and the mean of the times at which a celestial object is observed on each wire, is the instant of its being on the meridian, if the time by the clock is correct.

This is one of the most important instruments in an observatory, as with it and a sidereal clock, the right ascensions of all celestial objects are determined (for when an object is on the meridian, the sidereal time at the place of observation is that object's right ascension). The principal adjustments of this instument are, to level the axis, to place the middle wire perpendicular to the horizon, and the line of collimation, or line of sight, perpendicular to the horizon axis. For making these adjustments the following rules* may suffice:—

1. Slide the eye-piece in or out till the wires are distinctly seen. Direct the telescope to some distant and well-defined object, and turn the milled head on the side of the transit, till the object is seen with perfect distinctness. Place the level on the axis, and bring the bubble to the middle by a screw, which elevates or depresses one of the Ys or supports. The axis will then be parallel to the horizon.

2. When the middle wire covers some well-defined object, take the instrument out of the supports, and carefully invert it, when if the wire no longer covers the same part of the

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* By Captain Kater, R.N.

object, correct half the error by means of the screws which act horizontally on the wires, and the remaining half by those which act on the supports.

To observe the Transit of a Celestial Object.

At the side of the instrument are circles which mark altitude. From the Nautical Almanac find what bright stars are about to pass the meridian, by observing the right ascensions and the time by the sidereal clock ; the latter will show when the star is about to pass the meridian. Find the altitude of the star, by applying the declination to the latitude of the place of observation in the reverse way to that given in Chapter II.; the result is the zenith distance, and the complement of the zenith distance is the altitude : set the instrument to this altitude, the appearance of the star may then be looked for in the instrument.

If the transit of the Sun is to be observed, the instrument should be put in the plane of the meridian (if not already so fixed) before 12 a.m., and set to the meridian altitude.

At the transit across each wire, the time is marked by a second person ; from the chronometer if the object be the Sun, and from the sidereal clock if a star.

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